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Plenary Talks

Recent advances in unfitted finite element methods

Santiago Badia

Monash University

In this talk, I will give an overview of the last advances in unfitted finite element techniques for the numerical approximation of partial differential equations. Standard finite element methods (FEMs) require cumbersome and time-consuming body-fitted mesh generation. Conversely, unfitted FEMs provide a great amount of flexibility at the geometrical discretisation step. They can embed the domain of interest in a geometrically simple background grid (usually a uniform or an adaptive Cartesian grid), which can be generated and partitioned much more efficiently. Analogously, they can easily capture embedded interfaces. As a result, unfitted FEMs are generating interest in applications with moving interfaces and varying domains. However, naive unfitted methods lead to unstable and severe ill-conditioned discrete problems, unless a specific technique mitigates the problem. Different techniques have been developed so far, which rely on perturbation (stabilisation) of the problem itself, or a redefinition of finite element spaces based on aggregation meshes and discrete extension operators. We will describe the main challenges and methods. We will show links between different approaches and their effectiveness. We will cover topics like space-time discretisations, moving interfaces, adaptive refinement and high-contrast interface problems. We will also discuss the geometrical discretisation and integration steps in the unfitted workflow. Numerical analysis results, experiments, and implementation aspects will be discussed.

Santiago Badia is a Distinguished Professor of Computational Mathematics at Monash University (Melbourne, Australia). Previously, he served as a Professor at UPC (Barcelona, 2009–2019) and as a researcher at Sandia National Labs (Albuquerque, 2007–2008) and Politecnico di Milano (2006). His research advances finite element methods in singularly perturbed regimes, indefinite systems, multiphysics, and multiscale problems. He has made notable contributions to stabilized and unfitted finite element methods, large-scale parallel domain decomposition solvers, and coupling techniques, authoring over 100 publications in these areas. He also leads open-source scientific software projects, including Gridap. Prof. Badia has earned numerous prestigious awards, such as the 2016 Agustin de Betancourt Award (Royal Academy of Engineering, Spain), the ICREA Academia Award (Catalonia), the 2012 Young Researcher Award in Applied Mathematics (Spain), the Juan Carlos Simo Award (Spain), the 2006 ECCOMAS PhD Award (Europe), and the 2006 SEMNI Award for the best PhD Thesis in Computational Mechanics (Spain). He has led 13 major research projects across Australia, Spain, and Europe, including an ERC Starting Grant, two ERC Proof of Concept Grants, two Marie Curie Fellowships, and two ARC Discovery Projects.

Mixed precision matrix computations

Erin Carson

Charles University

Support for arithmetic in multiple precisions and number formats is becoming increasingly common in emerging architectures. Mixed precision capabilities are already included in many machines on the TOP500 list and will be a crucial hardware feature going forward. From a computational scientist's perspective, our goal is to determine how and where we can safely exploit mixed precision computation in our codes to improve performance. This requires both an understanding of performance characteristics as well as a rigorous understanding of the theoretical behavior of algorithms in finite precision arithmetic.

We discuss the challenges of designing mixed precision algorithms and give three cases where low precision can often safely be used to improve performance. One such case, common in computational science, is when there are already other significant sources of "inexactness" present, e.g., discretization error, measurement error, or algorithmic approximation error. In this instance, analyzing the interaction of these different sources of inexactness can give insight into how the finite precision number formats should be chosen in order to "balance" the errors, potentially improving performance without a noticable decrease in accuracy. We present a few recent examples of this approach, which demonstrate the potential for the use of mixed precision in numerical linear algebra.

Erin Carson is an Associate Professor at the Faculty of Mathematics and Physics at Charles University. Her research involves the analysis of matrix computations and the development of parallel algorithms for large-scale settings, with a particular focus on their finite precision behavior on modern heterogeneous hardware. She currently serves as Secretary of the SIAM Activity Group on Supercomputing, as the co-chair of the GAMM Activity Group on Applied and Numerical Linear Algebra, as an Associate Editor of ACM TOPC and SIAM SIMAX, and as a member of the EuroHPC Joint Undertaking Access Resource Committee. She is the recipient of the 2025 Wilkinson Prize in Numerical Analysis and Scientific Computing.

Fast boundary element methods beyond homogeneous media: towards realistic wave propagation modeling

Stéphanie Chaillat

CNRS – Laboratoire POems

Boundary Element Methods (BEMs), based on the discretization of boundary integral equations, have proven particularly well-suited for modeling wave propagation in unbounded domains. In recent years, significant advances have been made within the BEM community to make these methods applicable to realistic configurations. Fast algorithms—such as the Fast Multipole Method, low-rank approximations, and mesh adaptivity—have been developed to overcome the memory limitations inherent to BEM, particularly the dense nature of the system matrix. These advances have also significantly reduced computational costs, making BEM a competitive option for large-scale simulations. While fast BEMs are now mature and effective for problems involving homogeneous media and simple geometries, their applicability remains limited in more complex scenarios. In this talk, I will discuss two directions to extend the scope of BEM-based methods. First, I will explore how concepts from volumetric domain decomposition methods can inspire new coupling strategies between FEM and BEM for multiphysics problems. Second, I will present recent developments based on the multi-trace formalism, inspired by domain decomposition methods, which enable the modeling of piecewise homogeneous domains. These approaches will be illustrated through numerical results that highlight their potential for addressing challenging wave propagation problems in both academic and industrial contexts.

Stéphanie Chaillat is a CNRS Research Director and a member of the POEMS team (a joint research unit between CNRS, INRIA, and ENSTA Paris). She received her PhD in computational mechanics from École des Ponts in 2008, followed by a postdoctoral position at the College of Computing at the Georgia Institute of Technology in Atlanta. She joined CNRS in 2010. Her research lies in the field of numerical simulation of wave propagation, with a particular focus on the development of fast and accurate methods, especially boundary element methods (BEMs). She is interested in the modeling of realistic wave phenomena with significant scientific, industrial, and environmental impact, such as seismic waves, underwater acoustics or fluid-structure interactions. Her work combines mathematical modeling, numerical analysis, and algorithmic implementation.

Domain decomposition for molecular dynamics

Björn Engquist

The University of Texas Austin

We will first review molecular dynamics based on empirical potentials and discuss domain decomposition for distributed computing as is practiced in the computational molecular dynamics' community. The domain decomposition is done in space or time domain but also in probability. Then we will focus on milestoning, which is a computational methodology introduced by Ron Elber. Milestoning is a domain decomposition strategy that aims at reducing the overall computational complexity. The goal is to be able to simulate processes that occur over relatively long time as, for example, protein folding. The domain boundaries are hare called milestones and the coupling is in between fluxes and sources. We assume a stochastic model and analyze the somewhat unorthodox domain coupling via the related Fokker-Planck equation.

Björn Engquist is a Professor in the Department of Mathematics at the University of Texas at Austin, holding the Computational and Applied Mathematics Chair I and he is Director of the Center for Numerical Analysis at the Oden Institute. He earned his Ph.D. in numerical analysis from Uppsala University in 1975. Throughout his career, Engquist has held professorships at the University of California, Los Angeles (UCLA), Uppsala University, the Royal Institute of Technology (KTH) Stockholm and Princeton University, where he served as the Michael Henry Stater University Professor of Mathematics and Applied and Computational Mathematics. He was also the Director of the Research Institute for Industrial Applications of Scientific Computing and the Centre for Parallel Computers at KTH.

Engquist's research focuses on the development, analysis, and application of numerical methods for differential equations. His work includes the development of absorbing boundary conditions, nonlinear high-resolution schemes for compressible fluid dynamics, computational multiscale methods, fast numerical algorithms for wave propagation and applications of optimal transport to inverse problems in seismology.

Engquist has received several honors and awards. He is a member of the American Academy of Arts and Sciences, the Royal Swedish Academy of Sciences, the Royal Swedish Academy of Engineering Sciences, and the Norwegian Academy of Science and Letters. He was awarded the first SIAM James H. Wilkinson Prize in Numerical Analysis and Scientific Computing in 1982, the Celsius medal gold 1992, the Peter Henrici Prize 2011, the George David Birkhoff Prize 2012, the Pioneer Prize 2015 and College de France Medal 2018. He has twice been invited speaker at the International Congress of Mathematicians.

Fast high-order solvers on simplices for the de Rham complex

Patrick Farrell

University of Oxford

We present new finite elements for solving the Riesz maps of the de Rham complex on triangular and tetrahedral meshes at high order. The finite elements discretize the same spaces as usual, but with different basis functions, so that the resulting matrices have desirable properties. These properties mean that we can solve the Riesz maps to a given accuracy in a p-robust number of iterations with $\mathcal{O}(p^6)$ flops in three dimensions, rather than the naive $\mathcal{O}(p^9)$ flops.

The degrees of freedom build upon an idea of Demkowicz et al., and consist of integral moments on an equilateral reference simplex with respect to a numerically computed polynomial basis that is orthogonal in two different inner products. As a result, on the reference equilateral simplex, the resulting stiffness matrix has diagonal interior block, and does not couple together the interior and interface degrees of freedom. Thus, on the reference simplex, the Schur complement resulting from the elimination of interior degrees of freedom is simply the interface block itself.

This sparsity is not preserved on arbitrary cells mapped from the reference cell. Nevertheless, the interior-interface coupling is weak because it is only induced by the geometric transformation. We devise a preconditioning strategy by neglecting this interior-interface coupling. We precondition the interface Schur complement with the interface block, and simply apply point-Jacobi to precondition the interior block. The combination of this approach with a space decomposition method on vertex and edge star patches allows us to efficiently solve the canonical Riesz maps at very high order.

Joint work with P.D. Brubeck, R.C. Kirby, and C. Parker.

Patrick Farrell is a Professor in the numerical analysis group at the University of Oxford, and (for 2025-2026) the Donatio Universitatis Carolinae Chair in the faculty of mathematics and physics at Charles University in Prague. His research interests are in the numerical solution of partial differential equations arising in physics and chemistry.

He obtained his bachelor's degree in mathematics from the University of Galway, and his doctorate from Imperial College London in 2010. His doctoral thesis won the Roger Owen prize from the UK Association for Computational Mechanics, and the Janet Watson prize from Imperial.

He has been awarded an EPSRC Early Career Research Fellowship (2013-2018), the 2015 Wilkinson Prize for Numerical Software, second place in the 2015 Leslie Fox Prize in Numerical Analysis, the 2021 Charles Broyden Prize in optimisation, a 2021 Whitehead Prize from the London Mathematical Society, and the 2025 SIAM Germund Dahlquist Prize.

Higher-order locally implicit methods for linear Maxwell's equations

Marlis Hochbruck

Karlsruhe Institute of Technology

In this talk, we discuss the construction and analysis of higher-order time integration schemes for the full discretization of linear Maxwell equations on locally refined spatial grids with discontinuous Galerkin methods. Roughly speaking, we decompose the spatial domain into two subdomains, which we refer to as stiff and nonstiff subdomains, respectively. In the stiff subdomain, we gather all mesh elements which have a small diameter or a small material parameter (leading to stiff degrees of freedom). All other mesh elements are assigned to the nonstiff subdomain. We assume that the degrees of freedom in the stiff subdomain is considerably smaller than in the nonstiff subdomain.

The locally implicit scheme is based on a higher-order implicit method, e.g., an algebraically stable Runge–Kutta method like a Gauss collocation method. Our main contribution is to propose a preconditioned Krylov subspace method for solving the linear systems arising in each time step. The preconditioner is designed in such a way that its convergence only depends on the nonstiff subdomain but not on the stiff one. We will sketch a proof of this result using approximation theory in the complex plane using Faber polynomials. Finally, we verify our theoretical findings by numerical experiments.

This approach is applicable to any implicit scheme and also works for exponential integrators, where the action of the matrix exponential is approximated by rational Krylov subspace methods. It is even applicable to nonlinear problems, where such linear systems arise within the Newton iterations.

Reference:

M. Hochbruck, J. Köhler, and P. M. Kumbhar. Preconditioned implicit time integration schemes for Maxwell's equations on locally refined grids. CRC 1173 Preprint 2022/29, Karlsruhe Institute of Technology, 2022. URL https://doi.org/10.5445/IR/1000148078.

Marlis Hochbruck is a professor in the numerical analysis group at Karlsruhe Institute of Technology (KIT) in Germany since 2010. Previously, she was a professor at Heinrich-Heine University Düsseldorf (1998-2010), a postdoctoral researcher at the University of Tübingen (1994-1998) and the University of Würzburg (1992-1994), and a doctoral researcher at University of Karlsruhe (1989-1992).
Her research interests are time integration of partial differential equations, computational physics, and Krylov subspace methods for linear systems and matrix functions.
She served in several committees of the German Research Foundation (DFG), e.g., from 2014-2021 as one of the vice presidents. Since 2015, she is the spokesperson of the Collaborate Research Center "Wave phenomena: analysis and numerics" (www.waves.kit.edu) funded by DFG. In addition she is currently a research integrity officer at KIT.

Robust overlapping Schwarz methods and their applications

Pierre Jolivet

Sorbonne Université, CNRS, LIP6

Recent advances in domain decomposition preconditioners have made it possible to handle large linear systems of increasing complexity. In this presentation, I will give some insight into how they are efficiently implemented in high-level libraries such as PETSc and HPDDM. These implementations allow domain specialists to perform large-scale analyses that were previously difficult to deal with, even with state-of-the-art solvers.

Pierre Jolivet is a research scientist at the French National Centre for Scientific Research (CNRS) affiliated with the Laboratoire d'Informatique de Paris 6 (LIP6) at Sorbonne Université. His research focuses on high-performance computing particularly in developing fast and robust solvers for computational sciences. He is an active contributor to various open-source libraries such as PETSc, FreeFEM, and HPDDM.

Training of deep neural networks using multilevel and domain-decomposition methods

Alena Kopanicakova

University of Toulouse

Training deep neural networks (DNNs) is predominantly carried out using stochastic gradient method and its variants. While these methods are robust and widely applicable, their convergence often deteriorates for large-scale, ill-conditioned, or stiff problems commonly encountered in scientific machine learning. This has motivated the development of more advanced training strategies that can accelerate convergence, offer better parallelism, enable convergence control, and facilitate the automatic tuning of hyperparameters. To this end, we will introduce a novel training framework for DNNs inspired by nonlinear multilevel and domain-decomposition (ML-DD) methods. Starting from deterministic ML-DD algorithms, we will discuss how to ensure convergence in the presence of the subsampling noise. Moreover, we will present several strategies for constructing a hierarchy of subspaces by exploring the properties of the network architecture, data representation, and the loss function. The numerical performance of the proposed ML-DD training algorithms will be demonstrated through a series of numerical experiments from the field of scientific machine learning, such as physics-informed neural networks and operator learning approaches.

Alena Kopaničáková is an Associate Professor at Toulouse-INP (ENSEEIHT) and a member of the Parallel Algorithms and Optimization (APO) team at the IRIT Laboratory. She is also affiliated with the Artificial and Natural Intelligence Toulouse Institute (ANITI), where she holds an international research chair focused on the hybridization of AI and large-scale numerical simulations for engineering design. Prior to her appointment in Toulouse, she was a postdoctoral researcher at Brown University (USA) and at the Università della Svizzera italiana (Switzerland), where she also completed her PhD. Her research interests span nonlinear multilevel optimization, domain decomposition methods, scientific machine learning, hybrid (AI-augmented) iterative methods, phase-field modeling of fracture, and scientific software development.

Domain Decomposition and Beyond

Jan Mandel

University of Colorado

This lecture offers a personal perspective on making early contributions to a field — and choosing not to remain in it. With a focus on the formative phase of Domain Decomposition, I will aim to acknowledge the colleagues who provided its essential theoretical foundations, practical motivations, and key insights. I will also touch on related areas I contributed to that shared similar core principles, such as Iterative Aggregation, Multigrid Methods, and the p-version Finite Element Method, as well as later distinct work on coupled Wildfire and Weather Modeling, Probability, Data Science, and Machine Learning.

Rather than presenting a technical update, I will try to elucidate the driving ideas and illustrate a recurring strategy across these diverse areas: identifying gaps, selecting problems of interest to prospective users, developing methods with usability in mind, and seeking opportunities for analysis and proofs. In many cases, I moved on once others had taken up the most compelling directions. This talk will reflect on that pattern — not as a suggestion to follow, but as one way of contributing.

Jan Mandel received his education in Informatics, Mathematical Methods in Economics, and Numerical Mathematics at the Faculty of Mathematics and Phycis, Charles University, Prague, Czechoslovakia. Since 1986, he has been faculty at the University of Colorado Denver. Now Professor Emeritus, he continues his NASA research, and he is also HPC system administrator.

Efficient space-time methods for solving wave propagation challenges

Ilario Mazzieri

Politecnico di Milano

The numerical simulation of wave propagation presents significant challenges, particularly when dealing with complex geometries, heterogeneous media, and high-frequency regimes. Traditional time-stepping methods often struggle with achieving high-order accuracy while maintaining computational efficiency. This talk explores recent developments in domain decomposition methods for hyperbolic problems, focusing on space-time Restricted Additive Schwarz (XT-RAS) techniques. This approach provides a parallelizable framework that enhances computational performance while preserving high-order accuracy in both space and time. We analyze convergence in both continuous and discrete settings and investigate how time-windowing and time-integration schemes affect stability and performance. We then introduce pipeline and adaptive XT-RAS strategies, enabling parallelism in space and time. Numerical experiments support theoretical insights, and connections with tent-pitching approaches are discussed as a promising framework for parallel time integration.

Ilario Mazzieri is Associate Professor of Numerical Analysis at the MOX Laboratory, Politecnico di Milano where he obtained a PhD in Mathematical Models and Methods in Engineering (cum laude, Doctor Europaeus). His research focuses on high-order numerical methods for wave propagation, including elastodynamics, acoustics, and multiphysics problems. He is the author of over 35 peer-reviewed journal articles and an invited speaker at numerous international conferences. He has coordinated and participated in many national and EU-funded research projects (PRIN, ERC, CINECA). He is the lead developer of the seismic simulation code SPEED and co-author of the LYMPH software for polytopal methods. His work combines theoretical analysis, high-performance computing, and practical impact in geophysics and engineering.

Preconditioning, weighting and deflation applied to non-symmetric linear systems

Nicole Spillane

CNRS – Ecole polytechnique

This talk considers the solution of non-symmetric linear systems by GMRES. The objective is to find out, both, how to predict the convergence of GMRES and how to accelerate it. To make a parallel with symmetric positive definite (spd) linear systems, we would like the non-spd equivalent of the statement that "a good preconditioner is a preconditioner that reduces the condition number''. Three different accelerators are considered and combined: preconditioning, deflation, and weighting. Weighting, the lesser-known technique, consists in changing the inner product in which the GMRES algorithm operates. In cases where the problem matrix is positive definite, it is shown that applying a symmetric preconditioner H can result in a convergence bound that depends only on how well H preconditions the symmetric part of A and on how non-symmetric the problem is. This already leads to a strategy to design scalable preconditioners by domain decomposition. Convergence is accelerated further by deflating the high-frequency vectors of a well-chosen generalized eigenvalue problem. Numerical illustrations provide backup for our findings.

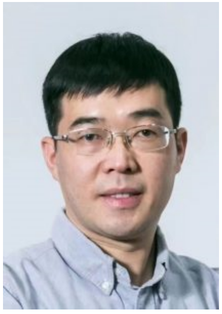
Nicole Spillane is a researcher at the French National Center for Scientific Research (CNRS). Her laboratory is the Center for Applied Mathematics of Ecole polytechnique in Paris. As an applied mathematician, Nicole's focus is on the analysis, development and application of large scale linear solvers. During her PhD at Université Paris Sorbonne, she participated in developing the GenEO coarse space in domain decomposition. She received two best PhD awards from AMIES and CSMA. She went on to propose the adaptive multipreconditioned conjugate gradient algorithm. In connection with this work, in 2017, she received the Leslie Fox Prize in Numerical Analysis from the UK's Institute of Mathematics and its Applications. Her current interests are in applying domain decomposition methods to PDEs with stochastic coefficients, and on solving non-symmetric linear systems. She is the Principal Investigator of ANR DARK on Domain Decomposition Accelerators for Robust Krylov Subspace Methods. She has also begun to explore the efficient simulation of PDEs on quantum computers.

Explorations in developing intelligent AMG solver for sequences of large-scale sparse linear systems

Xiaowen Xu

Institute of Applied Physics and Computational Mathematics – Beijing

Solving sequences of large-scale sparse linear systems is a critical performance bottleneck in many practical applications. The primary challenge comes from the dynamically changing characteristics of sparse matrices within the sequence, making it almost impossible for any fixed algorithmic strategy to achieve optimal performance for all systems in the sequence. Given an application scenarios, how to design an intelligent solvers with automatic tuning capability has become a crucial concern in practical application, and its core is to automatically achieve the optimal mapping between matrix feature space and algorithm space. In this talk, we take the widely used AMG (algebraic multigrid) solver in practical applications as an example to introduce the exploration and practice of intelligent solvers. Based on the divide and conquer principle, using performance modeling and machine learning techniques, the AMG solver achieves automatic tuning at component-level and parameter-level. We will introduce the intelligent AMG framework and demonstrate its effectiveness in typical practical applications.



Xiaowen Xu is a Professor and deputy of Institute of Applied Physics and Computational Mathematics (IAPCM), Beijing, China. He got his B.S degree from Xiangtan University in 2002, and his PhD degree from Chinese Academy of Engineering Physics in 2007. His research interests include high performance numerical algorithms in scientific and engineering fields, he is mainly engaged in the development of parallel sparse linear solver for large-scale numerical simulation. He is one of the core developers of the parallel programming framework called JASMIN, which enables complex numerical simulations to run efficiently on modern supercomputers and is widely used in various scientific and engineering applications in China.

Minisymposia

MS01 – Domain decomposition methods for wave-type problems

Organizers: Roland Maier, Tim Buchholz, Gabriele Ciaramella

This minisymposium is about theoretical and computational aspects of domain decomposition methods for partial differential equations that model wave-type problems. Examples are classical wave equations, Schrödinger equations, Maxwell- and Helmholtz-type equations, which are critical in fields such as acoustics, electromagnetism, and quantum mechanics. Wave-type problems often pose unique challenges due to oscillatory nature, high-frequency behavior, or the need for accurate resolution over large computational domains. Domain decomposition methods offer a promising approach to tackle these challenges, enabling efficient parallelization, localized error control, and scalable algorithms for large-scale simulations. In this minisymposium, we aim to bring together experts that work on different aspects of these research question, including recent theoretical results, innovative algorithms, and their applications.

List of Speakers

- C. Alber (Heidelberg University) – *Two-level Schwarz preconditioning based on multiscale spectral generalized FEM for heterogeneous Helmholtz problems*.
- M. Bonazzoli (Inria, Institut Polytechnique de Paris) – *Multi-domain FEM-BEM coupling for acoustic scattering*.
- T. Buchholz (Karlsruhe Institute of Technology) – *A non-iterative domain decomposition time integrator for linear wave equations*.
- G. Cicalese (Politecnico di Milano) – *Optimized red–black waveform relaxation for the damped wave equation*.
- M. Corti (Politecnico di Milano) – *Discontinuous Galerkin discretizations of Fisher-Kolmogorov with application to neurodegenerative diseases*.
- D. Gallistl (University of Jena) – *Localized implicit time stepping for the wave equation*.
- M.J. Gander (University of Geneva) – *Damping in hyperbolic and time harmonic wave propagation problems and its influence on domain decomposition solvers*.
- M. Grote (University of Basel) – *Explicit local time-stepping methods for the wave equation with optimal convergence*.
- B. Martin (University of Liège) – *To overlap or not to overlap? A large-scale investigation for Helmholtz problems with multiple sources*.
- I. Mazzieri (Politecnico di Milano) – *Schwarz waveform relaxation and the unmapped tent-pitching method in 3D*.
- H. Zhang (Xi'an Jiaotong-Liverpool University) – *Numerical investigation on some coarse problem for wave propagation*.

Two-level Schwarz preconditioning based on multiscale spectral generalized FEM for heterogeneous Helmholtz problems

Christian Alber

Heidelberg University

We present and analyze a two-level restricted additive Schwarz (RAS) preconditioner for heterogeneous Helmholtz problems, based on a multiscale spectral generalized finite element method (MS-GFEM) proposed in [2]. The preconditioner uses local solves with impedance boundary conditions, and a global coarse solve based on the MS-GFEM approximation space constructed from local eigenproblems. It is derived by first formulating MS-GFEM as a Richardson iterative method, and without using an oversampling technique, reduces to the preconditioner recently proposed and analyzed in [1].

We prove that both the Richardson iterative method and the preconditioner used within GMRES converge at a rate of Λ under some reasonable conditions, where Λ denotes the error of the underlying MS-GFEM approximation. Notably, the convergence proof of GMRES does not rely on the 'Elman theory'. An exponential convergence property of MS-GFEM, resulting from oversampling, ensures that only a few iterations are needed for convergence with a small coarse space. Moreover, the convergence rate Λ is not only independent of the fine-mesh size h and the number of subdomains, but decays with increasing wavenumber k. In particular, in the constant-coefficient case, with $h \sim k^{-1-\gamma}$ for some $\gamma \in (0,1]$, it holds that $\Lambda \sim k^{-1+\frac{\gamma}{2}}$. We present extensive numerical experiments to illustrate the performance of the preconditioner, including 2D and 3D benchmark geophysics tests, and a high-contrast coefficient example arising in applications.

References

[1] Q. Hu and Z.Li. A novel coarse space applying to the weighted Schwarz method for Helmholtz equations. *arXiv* (2024).

[2] C. Ma, C. Alber, and R. Scheichl. Wavenumber explicit convergence of a multiscale generalized finite element method for heterogeneous Helmholtz problems. *SIAM. J. Numer. Anal.* (2023).

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Multi-domain FEM-BEM coupling for acoustic scattering

Marcella Bonazzoli

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We model time-harmonic acoustic scattering by an object composed of piece-wise homogeneous parts and an arbitrarily heterogeneous part. We propose and analyze new formulations that couple, adopting a Costabel-type approach, boundary integral equations for the homogeneous subdomains with volume variational formulations for the heterogeneous subdomain. This is an extension of the Costabel FEM-BEM coupling to a multi-domain configuration, with cross-points allowed, i.e. points where three or more subdomains are adjacent. While generally just the exterior unbounded subdomain is treated with the BEM, here we wish to exploit the advantages of BEM whenever it is applicable, that is, for all the homogeneous parts of the scattering object. Our formulation is based on the multi-trace formalism, which initially was introduced for acoustic scattering by piece-wise homogeneous objects. Instead, here we allow the wavenumber to vary arbitrarily in a part of the domain. We prove that the bilinear form associated with the proposed formulation satisfies a Gårding coercivity inequality, which ensures stability of the variational problem if it is uniquely solvable. We identify conditions for injectivity and construct modified versions immune to spurious resonances.

Joint work with X. Claeys.

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A non-iterative domain decomposition time integrator for linear wave equations

Tim Buchholz

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In this talk we will motivate, construct and analyze a non-iterative domain decomposition time integrator for the linear wave equation.

The central concept involves combining an implicit solution step on spatial subdomains with a cost-effective, yet precise, local, explicit prediction step. In that sense the method is similar to the methods from Blum, Lisky and Rannacher or Dawson and Dupont, where mainly parabolic problems were considered. While the locality of the prediction combined with the decomposition into subdomains allows for parallelization across space, the scheme progresses sequentially through time. However, unlike similar methods, this time-stepping is only performed once, without any iterations involved. Moreover, we show that the scheme achieves second-order convergence in time.

An interesting extension of this method can be achieved by combining it with a method by Gallistl and Maier, in which we replace the explicit prediction step with a localized implicit one.

We conclude the presentation with numerical experiments.

Joint work with M. Hochbruck and R. Maier.

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Optimized red–black waveform relaxation for the damped wave equation

Gerardo Cicalese

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We present an optimized Red–Black Waveform Relaxation (WR) method for the one-dimensional damped wave equation with combined *telegrapher damping* ($\partial_t u$) and *viscoelastic damping* ($\partial_t \partial_{xx} u$). We leverage domain decomposition theory for hyperbolic–parabolic systems through three key components: (1) parallel decomposition across N overlapping subdomains, (2) red–black partitioning to alternate subdomain updates, and (3) optimized Robin transmission conditions at each interface.

We combine frequency-domain analysis with time-aware error bounds derived from Green's-function kernels, explicitly investigating how the damping parameters influence convergence. We demonstrate through numerical experiments that our optimized interface conditions reduce iterative error substantially faster than classical Dirichlet coupling and confirm the method's efficacy for wave-dominated systems with mixed dissipation mechanisms.

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Discontinuous Galerkin discretizations of Fisher-Kolmogorov with application to neurodegenerative diseases

Mattia Corti

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Neurodegenerative diseases have a significant global impact, affecting millions of individuals worldwide. Some of them, known as proteinopathies (for example, Alzheimer's and Parkinson's diseases), are characterized by the accumulation and propagation of toxic proteins known as prions. Mathematical models of prion dynamics play a crucial role in understanding disease progression. Several models have been proposed to describe the misfolding process with various levels of detail, the simplest but most used in literature is the Fisher-Kolmogorov. Typically, the Fisher-Kolmogorov problem in neurodegenerative problems exhibits a travelling wave solution. The construction of the classical discontinuous Galerkin formulation causes a lack of positivity preservation of the numerical solution, which loses its physical meaning. This talk presents a structure-preserving, high-order, unconditionally stable numerical method for approximating the solution to the Fisher-Kolmogorov equation on polytopic meshes. The model problem is reformulated using an entropy variable to guarantee solution positivity, boundedness, and satisfaction of a discrete entropy-stability inequality at the numerical level. Moreover, we performed simulations of alpha-synuclein propagation in a two-dimensional brain geometry segmented from MRI data, providing a relevant computational framework for modeling synucleopathies.

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Localized implicit time stepping for the wave equation

Dietmar Gallistl

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This contribution proposes a discretization of the acoustic wave equation with possibly oscillatory coefficients based on a superposition of discrete solutions to spatially localized subproblems computed with an implicit time discretization. Based on exponentially decaying entries of the global system matrices and an appropriate partition of unity, it is proved that the superposition of localized solutions is appropriately close to the solution of the (global) implicit scheme. It is thereby justified that the localized (and especially parallel) computation on multiple overlapping subdomains is reasonable. Moreover, a re-start is introduced after a certain amount of time steps to maintain a moderate overlap of the subdomains. Overall, the approach may be understood as a domain decomposition strategy in space on successive short time intervals that completely avoids inner iterations.

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Damping in hyperbolic and time harmonic wave propagation problems and its influence on domain decomposition solvers

Martin J. Gander

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The second order wave equation is the prototype hyperbolic problem one sees in a first course on partial differential equations, as a model for the vibrating string. If one uses this model however to model a string of a guitar or a piano, one discovers that the string modeled does not oscillate at all like it should, its shape does not correspond to the typical sine function shape one sees on these instruments. The reason is that the hyperbolic model is too crude an approximation of the string. Damping is missing, and it is not just the damping that makes the amplitude decrease, like in the telegraphers equation, it is more importantly the viscoelastic damping. Only with such damping the model corresponds to what we observe in reality.

Nevertheless hyperbolic and time harmonic wave propagation problems are often used without damping, and this makes them hard to solve by iterative methods. Since the introduction of the shifted Laplace preconditioner for Helmholtz problems, it is well known that damping can greatly help iterative solvers to perform well when solving time harmonic problems where they usually struggle. In nature, there is always damping, and the mathematical models without damping are approximations. I will present in my talk the influence of naturally present damping on the performance of Schwarz methods when solving damped Helmholtz problems, compared to the undamped case, and show quantitatively the big impact on Schwarz methods of such damping: even the very hard cavity case becomes easy to solve with impedance Schwarz methods if natural damping is present. I will also discuss the influence of damping in the time dependent case for a recent class of space time parallel Schwarz waveform relaxation methods called unmapped tent pitching.

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Explicit local time-stepping methods for the wave equation with optimal convergence

Marcus Grote

University of Basel

Adaptivity and local mesh refinement are crucial for the efficient numerical simulation of wave phenomena in complex geometry. Local mesh refinement, however, can impose a tiny time-step across the entire computational domain when using explicit time integration. By taking smaller time-steps but only inside locally refined regions, local time-stepping methods overcome the stringent CFL stability restriction imposed on the global time-step by a small fraction of the elements without sacrificing explicitness. Moreover, they always lead to optimal L^2-error estimates under a CFL stability condition independent of the coarse-to-fine mesh ratio. They also yield optimal H^1-convergence when the source vanishes across the coarse-to-fine mesh interface. To achieve optimal H^1-convergence even when the source is nonzero at the interface, we introduce a weighted transition defined through a discrete mesh-based distance function.

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To overlap or not to overlap? A large-scale investigation for Helmholtz problems with multiple sources

Boris Martin

University of Liège

In the context of inverse scattering problems such as Full Waveform Inversion (FWI), wave propagation must be simulated in an heterogeneous medium with a variety of source locations. In the frequency domain, this means solving the same PDE with multiple right-hand sides.

While traditionally solved with sparse direct solvers, large problems (e.g. at high frequency) become exorbitant to solve, in both time and memory requirements. For these reasons, domain decomposition methods are a popular alternative.

Both overlapping and non-overlapping methods are now mature for the Helmholtz equation, but choosing between both approaches in real conditions is not trivial, as many parameters are involved. In this talk, we present our optimized implementations of the Optimized Restrictive Additive Schwarz (ORAS) method as well as of non-overlapping Optimized Schwarz Methods (OSM). Particular care was taken to have efficient handling of the multiple sources. We perform large-scale comparisons on the LUMI supercomputer to solve systems with hundreds of millions of unknowns to get some insights into the strengths and weaknesses of each approach.

Joint work with C. Geuzaine and P. Jolivet.

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Schwarz waveform relaxation and the unmapped tent-pitching method in 3D

Ilario Mazzieri

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In this work, we present a novel equivalence between Mapped Tent Pitching (MTP) and Schwarz Waveform Relaxation (SWR), leading to the formulation of the Unmapped Tent-Pitching (UTP) method in three dimensions. We analyze UTP applied to the second-order wave equation, proving its convergence and characterizing the resulting 4D space-time tent structures. We show that, under suitable conditions, the advancing fronts generated by UTP and SWR coincide, and provide an explicit description of MTP's polytopes (tents) on a uniform 3D mesh. The proposed UTP algorithm is both conceptually simple and computationally effective. Moreover, UTP is naturally a space-time parallel algorithm for hyperbolic problems.

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Numerical investigation on some coarse problem for wave propagation

Hui Zhang

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The oscillatory waves require sufficient degrees of freedom to resolve. This restriction usually also applies to coarse problems for Schwarz methods. The resulting coarse problem is then too large. In other words, the coarse problem can not be too coarse, which prohibits a usual multilevel method. In this work, we investigate numerically an alternative approach to the coarse problem. The goal is to make the coarse correction as cheap as possible. First, we identify the slowly converging modes of the one-level Schwarz iteration. Then, we examine whether an alternative approach can efficiently remove those modes. A complexity analysis will be carried out.

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MS02 – Recent advancements in preconditioning techniques for high-performance computing systems

Organizers: Andrea Franceschini, Carlo Janna

High-Performance Computing (HPC) is increasingly essential for tackling complex simulations across diverse applications, from subsurface engineering to fluid dynamics, from structural mechanics to biomedicine. As the demand for realistic and high-resolution models grows, traditional numerical methods struggle under the pressures of computational cost and time efficiency, particularly in solving large-scale linear systems that can dominate simulation runtimes. This minisymposium seeks to highlight recent advancements in preconditioning techniques designed for next-generation HPC architectures, aiming to mitigate the challenges associated with the solution of extreme-size sparse linear systems ranging from hundreds of millions to billions of unknowns. Contributions will focus on innovative multigrid and multiscale preconditioners tailored for tackling the different numerical simulations. These advanced methods are crucial to improve the convergence rates of iterative solvers, particularly under the constraints of high fidelity and large problem sizes typical of modern industrial applications. Additionally, we will explore the implications of HPC developments, such as massively parallel computing and the utilization of graphical processing units (GPUs), on preconditioning strategies. Recent breakthroughs in the efficient utilization of memory hierarchies and optimized algorithm design will also be discussed, emphasizing their role in maximizing computational throughput. By bringing together experts and researchers in the field of numerical simulation, this minisymposium will provide a platform for exchanging ideas and sharing cutting-edge techniques aimed at advancing the frontiers of scalable linear solvers and their applications in real-world problems on exascale systems.

List of Speakers

- À. Alsalti-Baldellou (University of Padova) – *Aggressive coarsening for faster CFD simulations on GPU-accelerated supercomputers*.
- R. Falgout (Lawrence Livermore National Laboratory) – *New advances in HYPRE for semi-structured problems*.
- Z. Mammadov (Tartu University) – *GPU-accelerated FGMRES with overlapping Schwarz preconditioner for efficient solution of Helmholtz equations*.
- J. Schroder (University of Wuppertal, University of New Mexico) – *Structure preserving AMG coarsening and interpolation for PDE systems*.
- S. Thomas (AMD, Lehigh University) – *Streaming Gauss-Seidel for AMG coarse solves: a one-pass projection for Gram systems and spectral inclusion*.
- R. Tuminaro (Sandia National Laboratories) – *A new algebraic multigrid algorithm for H-curl problems based on energy minimization*.
- S. Van Criekingen (CNRS (IDRIS and Maison de la Simulation)) – *Two-level Schwarz methods using more subdomains than MPI processes: time and energy investigations*.
- C. Vuik (TU Delft) – *A scalable parallel solver for Helmholtz problems*.

Aggressive coarsening for faster CFD simulations on GPU-accelerated supercomputers

Àdel Alsalti-Baldellou

University of Padova

Owing to its effectiveness in solving elliptic PDEs, Algebraic Multigrid (AMG) is the default preconditioner of many Computational Fluid Dynamics (CFD) simulation codes. Nevertheless, its quality comes with relatively high memory requirements and setup and application costs, and making it lighter without harming its fast convergence yields significant speed-ups. A way to achieve this is through aggressive coarsening [1], which Chronos implements by computing the symbolic power of the adjacency matrix, T, corresponding to the graph of strongly connected unknowns (for a given strength-of-connection measure and filtering). Then, it computes a maximum independent set on the resulting matrix, T^k, which ensures there are no coarse nodes at a distance $\leq k$. The greater k, the more aggressive the coarsening and the fewer levels in the multigrid hierarchy, but the potentially worse accuracy and, therefore, slower convergence. Hence, to preserve AMG's excellent convergence, aggressive coarsening requires accurate prolongations relying on long-distance interpolations [2]. Chronos uses the dynamic-pattern least squares (DPLS) interpolation, which results in remarkably low operator complexities. Nevertheless, DPLS was designed to accommodate multiple test vectors, which makes it perform poorly on CFD problems due to their one-dimensional near-null space. However, combining it with energy minimization proved very effective in improving DPLS quality while keeping the operator complexity low, resulting in an AMG converging comparably regardless of the aggressive coarsening [3]. Numerical experiments harnessing our novel aggressive coarsening GPU implementation will be presented at the conference.

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New advances in HYPRE for semi-structured problems

Robert Falgout

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The HYPRE library is a suite of parallel linear solvers and preconditioners featuring multigrid methods. One of the hallmarks of the library is its conceptual linear system interfaces, which allows users to describe their linear systems in the traditional row-column based matrix format, but also in terms of (logically) structured grids and stencils. This semi-structured interface provides information that HYPRE can then exploit to gain performance. Although the semi-structured interface has been available for many years, users only had access to fully structured and fully unstructured multigrid methods, while practical applications often fall somewhere in between. Recently, the semi-structured component of HYPRE was overhauled and extended to allow for easier development of semi-structured solvers. In this talk, we discuss the new features and present details on a new semi-structured algebraic multigrid (SSAMG) method.

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GPU-accelerated FGMRES with overlapping Schwarz preconditioner for efficient solution of Helmholtz equations

Ziya Mammadov

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We are using GPU acceleration for the solution of subdomain problems of the overlapping Restricted Additive Schwarz method for the solution of the Helmholtz problem. We use the Flexible Generalised Minimal Residual (FGMRES) as the outer Krylov subspace method, allowing robust convergence of outer iterations with different preconditioners in each iteration.

Our target problem is the parallel solution of the high-frequency Helmholtz problem with absorption. Discretising this problem in domain decomposition setups results in large systems of linear equations with symmetric non-Hermitian matrices on each subdomain. This allows using a special version of the Conjugate Gradient method in their approximate solution.

We present an optimised implementation of the Conjugate Gradient method using PyOpenCL, where custom GPU kernels accelerate critical CG operations, including sparse matrix-vector multiplication, vector updates, dot products, and convergence checking. The implementation features GPU-based tolerance verification that enables early termination of subdomain solves if the desired accuracy level is reached. Our architecture leverages PyOpenCL's context management to minimise initialisation overhead and host-device communication, while supporting efficient multi-GPU execution through device-specific kernel compilation.

For the homogeneous problem, splitting a regular domain into identical overlapping subdomains with the same matrices is possible, enabling a GPU-efficient Block Conjugate Gradient approach for the simultaneous solution of multiple right-hand sides. Through the University of Tartu's HPC resources, we achieve parallel execution by distributing subdomains across multiple NVIDIA Tesla V100 GPUs. This approach demonstrates good scaling characteristics, with testing conducted for configurations of more than 100 subdomains, each containing over 250,000 unknowns.

Our experimental evaluation systematically assessed the performance of GPU-accelerated domain decomposition methods for solving the Helmholtz equation. Time complexity analysis revealed that our PyOpenCL-CG implementation achieved up to 6$\times$ speedup over traditional CPU-based solvers, particularly for large subdomains. We investigated the impact of subdomain overlap size, finding that convergence improved significantly when overlaps comprised 20% of the subdomain width, though larger overlaps introduced diminishing returns. Surprisingly, higher wavenumbers—typically associated with more challenging problem conditions—led to faster convergence, suggesting inherent conditioning benefits in certain regimes. These results provide practical insights for tuning domain decomposition parameters while highlighting the effectiveness of GPU acceleration for large-scale Helmholtz problems.

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Structure preserving AMG coarsening and interpolation for PDE systems

Jacob Schroder

University of Wuppertal, University of New Mexico

Algebraic multigrid (AMG) is a popular and effective solver for sparse linear systems arising from discretized PDEs. In this talk, we present an AMG method for PDE systems with large near-kernels (e.g., $H(\mathrm{curl})$ and $H(\mathrm{div})$) by building on compatible relaxation (CR) for coarsening and the generalized AMG (GAMG) perspective for interpolation construction. Thus, our main contributions concern our algebraic coarsening and interpolation approaches. For coarsening, we build a nodal dual problem which then allows us to algebraically coarsen nodes and edges in a structure preserving fashion using matching. For interpolation, we explore decoupling the fine and coarse variables to increase sparsity, so that in some cases, our method can reproduce re-discretization, even on unstructured meshes. We additionally explore prolongation smoothing as another automatic way of extending interpolation stencils. The relaxation procedure uses an automatic smoother construction, developed earlier by the authors, which finds local near kernel modes and constructs appropriate block relaxation schemes. Our numerical results indicate the effectiveness and promise of this two-grid approach for target Curl-Curl problems and the Stokes system.

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Streaming Gauss-Seidel for AMG coarse solves: a one-pass projection for Gram systems and spectral inclusion

Stephen Thomas

AMD, Lehigh University

We propose a matrix-free, GPU-native method for coarse-level correction in Algebraic Multigrid (AMG) that exploits the structure of Gram systems and spectral properties of Ritz approximations. By applying a single forward Gauss-Seidel iteration to the Gram system $P^T AP\alpha = P^T r$, where A is symmetric positive definite and P spans a Krylov or aggregation-based prolongation subspace, we construct an in-place A-norm projection equivalent to Modified Gram-Schmidt. This projection eliminates the need to form or invert the coarse matrix and replaces traditional triangular solves with a single streaming pass. Our approach is particularly well suited to modern GPU architectures, eliminating memory-intensive operations such as transpose access and explicit coarse matrix assembly.

Leveraging the Johnson and Horn eigenvalue inclusion theorem, we justify that the condition number $\kappa(P^T AP)$ improves with each level in the AMG hierarchy, making our one-pass method increasingly accurate and effective for coarse solves. This approach introduces a shift in AMG design: from matrix-centric to streaming-projection-centric coarse-level correction.

We demonstrate equivalence to A-norm projection via Gauss-Seidel splitting and prove that the forward sweep solution

$$\alpha^{(1)} = (I + L_s)^{-1} P^T r$$

approximates the exact Gram solution when $P^T AP \approx I + L_s + L_s^T$ and $\kappa(P^T AP)$ is small. We provide timing and convergence results on AMD MI300 and MI210 GPUs, comparing our method with traditional AMG coarse solvers and demonstrating order-of-magnitude improvements in memory bandwidth utilization.

Joint work with P. d'Ambra.

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A new algebraic multigrid algorithm for H-curl problems based on energy minimization

Raymond Tuminaro

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A new AMG algorithm is proposed for 1st order edge element discretizations of H-curl problems. The new algorithm leverages energy minimization AMG ideas to devise interpolation operators that satisfy a set of constraints while minimizing the energy of the associated grid transfer basis functions. Specifically, a standard AMG algorithm is applied to a related H-grad operator to first generate a nodal interpolation operator. Then, an energy minimization algorithm constructs a suitable H-curl edge interpolation operator. The key is to define constraints to enforce a commuting relationship associated with interpolation of a coarse nodal function followed by applying a fine level discrete gradient versus instead first applying a coarse discrete gradient followed by interpolating this to the fine level via edge interpolation. This new algorithm can be viewed as a generalization and improvement on earlier ideas by Reitzinger and Schoberl. It can be shown that satisfaction of this commuting relationship guarantees a coarse gradient operator lies within the null space of the coarse level curl-curl matrix, which is obtained by Galerkin projection using the edge interpolation operator. We will illustrate how the energy minimization algorithm can often generate "ideal" edge interpolation operators when supplied with "ideal" nodal interpolation. Numerical results will also be provided illustrating the overall efficacy of the approach.

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Two-level Schwarz methods using more subdomains than MPI processes: time and energy investigations

Serge Van Criekingen

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Typical supercomputing domain decomposition experiments use as many CPU processor cores (or CPUs for short), and as many MPI processes, as subdomains, each of the CPU corresponding to one MPI process and handling one subdomain. We here consider using more subdomains than CPUs, i.e., having each CPU handle more than one subdomain, and we show how this can be beneficial. Using more subdomains than MPI processes is a possibility given by PETSc, the library on which our code is based, in its (one-level) Restricted Additive Schwarz (RAS) implementation. We here consider RAS and optimized RAS (or ORAS) methods and, as coarse spaces, the Nicolaides coarse space which uses a constant basis function on each subdomain, and the merged coarse space previously developed by the authors, which uses a single coarse function at each cross point of the non-overlapping decomposition and is meant to be used with GMRES acceleration or with relaxation. We provide results in terms of time- as well as energy-to-solution, the latter metric being of increasing importance in supercomputing centers.

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A scalable parallel solver for Helmholtz problems

Cornelis Vuik

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A matrix-free parallel multi-level-deflation preconditioner combined with the Complex Shifted Laplacian preconditioner(CSLP) for the two-dimensional Helmholtz problems is presented. The Helmholtz problem, widely studied in seismic exploration, is hard to solve both in terms of accuracy and convergence, due to the scalability issues of the numerical solvers. For large-scale applications, high-performance parallel scalable methods are also indispensable. In our method, we use the preconditioned Krylov subspace methods to solve the linear system obtained from finite-difference discretization. The CSLP preconditioner is approximately inverted by one parallel geometric multi-grid V-cycle. Motivated by the observation that the eigenvalues of the CSLP-preconditioned system shift towards zero for large wavenumbers, deflation with multi-grid vectors and further high-order vectors were incorporated to obtain wave-number-independent convergence. We also compare Galerkin coarsening method and high-order re-discretization on the coarse grid. The matrix-vector products and the inter-grid operations are implemented based on the finite-difference grids without constructing the coefficient matrix. These adjustments lead to direct improvements in terms of memory consumption. Numerical experiments show that wavenumber independence has been obtained for medium wavenumbers. The matrix-free parallel framework shows satisfactory parallel performance and weak scalability.

Joint work with J. Chen and V. Dwarka.

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MS03 – Advanced discretization methods and solvers for coupled problems

Organizers: Denise Grappein, Tommaso Vanzan

The increasing complexity of modern scientific simulations has fueled research on novel discretization methods, which in turn require tailored solvers. This minisymposium focuses on coupled problems, addressing multi-physics, multi-scale, and multi-dimensional challenges. Presentations will cover topics such as high-order discretization schemes, reduced order models and the development of robust solvers capable of efficiently handling the interdependencies typical of coupled systems. Examples of applications include fluid-structure interaction problems, geophysical and biological processes and electromagnetism simulations.

List of Speakers

- S. Bertoluzza (CNR - IMATI "Enrico Magenes") – *An abstract framework for heterogeneous coupling: stability, approximation and applications*.
- B. Crippa (Politecnico di Milano) – *A mixed-dimensional model of the electrical treeing*.
- M. Discacciati (Loughborough University) – *Efficient coupling of local parametric Stokes and Darcy surrogate models via overlapping domain decomposition*.
- A. Franceschini (University of Padova) – *Advancements in mortar algorithms for non-conforming contact mechanics in geomechanical simulations*.
- D. Givoli (Technion - Israel Institute of Technology) – *Recent developments in the mixed-dimensional analysis of elastic structures*.
- Y. Liu (University of Macau) – *Solution-based coarse preconditioner for patient-specific blood flow simulations*.
- G. Teora (Politecnico di Torino) – *Modeling root water uptake in complex soils using the virtual element method*.
- W. Yu (Guangdong Provincial Key Laboratory of Interdisciplinary Research and Application for Data Science, BNU-HKBU United International College) – *Loosely coupled schemes for fluid-structure interactions*.

An abstract framework for heterogeneous coupling: stability, approximation and applications

Silvia Bertoluzza

CNR - IMATI "Enrico Magenes"

We introduce an abstract coupling framework reminiscent of FETI, for which we establish conditions for stability, minimal requirements for well-posedness both on the continuous and on the discrete level, as well as a stabilization strategy, acting only on the multiplier, that can be used to achieve stability of the discrete problem under very mild conditions. A general abstract recipe for the design of preconditioners is proposed, also inspired by FETI. Its application in the domain decomposition framework is discussed, resulting in both some known and some novel domain decomposition preconditioners.

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A mixed-dimensional model of the electrical treeing

Beatrice Crippa

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The electrical treeing is one of the main causes of degradation of the insulating components in the electric system, caused by the interaction between Partial Discharges and the polymeric surface, under the prolonged action of intense electric fields, and resulting in a typically highly branched gas-filled fracture in the material. This phenomenon can be described by a system of Partial Differential Equations [3], modeling the movement of charges within the defect and the evolution of the electric field and potential in both the gas and in the surrounding dielectric. A central limitation in the simulation of this phenomenon is the elevated computational cost, due to the need for very fine 3D meshes for numerical simulations on the branched and thin geometry of the electrical treeing. To overcome this obstacle, we approximate the treeing structure with a one-dimensional graph and derive a mixed-dimensional 3D-1D system of equations, significantly reducing the problem complexity. Our approach results in a 1D model describing the movement of charges in the gas, coupled with a 3D-1D electrostatic model [1]. The numerical solution of the mixed-dimensional electrostatic problem is based on mixed Finite Elements in the 3D domain, coupled with Finite Elements on the 1D graph, while for the 1D problem of charge concentrations we employ a time splitting, separating the chemical reactions from the diffusion and transport parts of the equations, solved with Finite Volumes on the 1D graph [2]. We validate the reduced model on simple geometries and apply it to a realistic electrical treeing structure, achieving a significant reduction in computational cost, and enabling simulations on complex geometries, where the generation of a full 3D mesh would be impractical.

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Efficient coupling of local parametric Stokes and Darcy surrogate models via overlapping domain decomposition

Marco Discacciati

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Parametric Stokes-Darcy systems can be encountered in the real-time computational design and optimisation of filtration devices, where parameters are used to characterise different operating conditions (e.g., inflow/outflow velocity and/or pressure, fluid viscosity, permeability, porosity, geometrical setting). While model order reduction techniques provide an established approach to solve parametric problems, the presence of two different physical models and of a possibly large number of parameters may cause the computational cost to become a bottleneck for practical applications. In this talk, we present a recent approach [1,2] to reduce the cost of constructing surrogate models for the parametric Stokes-Darcy system by computing local surrogate models in the offline phase using proper generalized decomposition (PGD). In the online phase of the algorithm, these local models are coupled via overlapping domain decomposition methods [3,4] in a way that is consistent with the underlying microscopic physical behaviour of the fluid. Numerical results to assess the accuracy, robustness and efficiency of the proposed methodology will be shown.

Joint work with B. Evans and M. Giacomini.

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Advancements in mortar algorithms for non-conforming contact mechanics in geomechanical simulations

Andrea Franceschini

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The accurate prediction of geomechanical behavior is crucial for the effective management of underground resources. This involves the simulation of various physical processes, such as fluid flow, poromechanics, fault activation, thermal transfer, and chemical reactions, which can occur simultaneously across different time and spatial scales. While significant progress has been made in analyzing and simulating individual subsurface processes, there remains active research aimed at coupling geomechanics with other relevant phenomena at appropriate scales, from both numerical and physical perspectives. One of the key challenges researchers face is coupling non-conforming subdomains. Traditional grid conformity requirements are too rigid, limiting the ability to capture local phenomena or apply different discretization schemes for different physical processes. For example, finite elements may be used on tetrahedral grids for momentum equations, while finite volumes are applied on hexahedral grids for mass balance or transport equations. To address this, we propose the use of mortar methods to couple different subdomains. The internal kernel of this approach has been simplified, allowing for efficient field interpolation between master and slave sides without the need for nonlinear projections. Additionally, this method is being extended to handle discontinuous mechanics, specifically to model fault and fracture behavior. This involves incorporating the standard Karush-Kuhn-Tucker (KKT) conditions, ensuring that sliding occurs only when a failure criterion is met, and enforcing non-compenetrability. The proposed methodology has been implemented within GReS, an open-source, modular platform designed to advance the development and prototyping of numerical algorithms for fully coupled, multi-physics, multi-domain geomechanical applications. GReS is built on a high-level programming platform (MATLAB), which lowers the entry barrier for new users and developers and simplifies the implementation and testing of innovative numerical algorithms. Its modular structure promotes contributions from developers at various levels, ranging from new physics and discretization schemes to algorithms for accelerating both linear and nonlinear solvers. This paper introduces the mortar algorithm and its extension to model non-conforming fractures, alongside the GReS platform and its current development status. Basic benchmarks will be presented to demonstrate the potential of the current algorithms, and future development plans will be discussed.

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Recent developments in the mixed-dimensional analysis of elastic structures

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A recurring theme in computational mechanics in recent years is the need to reduce the size of large discrete models. One type of such a reduction is spatial dimensional reduction, which one may perform in cases where the solution in some region of a high-dimensional (highD) computational domain, say two-dimensional (2D), behaves in a low-dimensional (lowD) way, say one-dimensional (1D).

The motivation in constructing a mixed-dimensional (say 2D-1D) model comes from the fact that solving the problem in its highD form everywhere may require a very large computational effort. The idea is thus to partly reduce the spatial dimension of the problem, in order to obtain a hybrid model which is much more efficient. One field of application where mixed-dimensional coupling is of special interest is that of the dynamics of elastic structures. Typically, the LowD model consists of the slender parts of the structure that have rod- or beam- or plate- or shell-like behavior, and which constitute most of the structure volume, while the HighD parts are the small regions that have to be modeled as 3D elastic bodies.

In the present work, we consider the 2D-1D, 3D-2D and 3D-1D coupling of models, to form a single hybrid model, for linear elastodynamics. We present some recent advances and new coupling methods. A special challenge is the attack of problems involving out-of-plane bending. For example, in the case where the highD model consists of 3D elastic elements, and the lowD model consists of 1D beam elements, there is a mismatch in the type of degrees of freedom (DOFs) between the two models, as the latter includes rotation DOFs and the former does not. Another aspect that makes this a difficult problem is the significant difference between the type of differential equations and finite elements (C0 vs. C1) used for each part of the problem. We will present several approaches for the coupling of such models.

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Solution-based coarse preconditioner for patient-specific blood flow simulations

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Numerical simulation of patient-specific blood flow problems is crucial for predicting and analyzing the behavior of blood flows, particularly in arterial networks with aneurysms or stenoses. These abnormalities often lead to complex flow patterns, challenging the convergence of the additive Schwarz preconditioner. The centerline-based coarse preconditioner, derived from a reduced one-dimensional model on the centerline, effectively captures principal flow features in healthy arteries but struggles with abnormal regions where flow patterns deviate significantly. To address this, in this talk, we introduce a novel two-level Schwarz preconditioner for these problems. Our solution-based coarse preconditioner leverages prior numerical solutions to extract both principal and local flow features, enhancing the robustness of the Schwarz preconditioner. Numerical experiments demonstrate its effectiveness for both normal and abnormal arteries, offering a useful tool for patient-specific simulations with various arteries.

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Modeling root water uptake in complex soils using the virtual element method

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Water uptake by plant roots plays a crucial role in the soil water balance. Mathematical models and simulations of the interactions between root systems and the surrounding soil provide valuable tools for understanding how plant roots influence soil water distribution and availability. This presentation introduces a novel optimization-based strategy for simulating the inherently challenging coupled problem of the interaction between a growing root system and the surrounding soil, with a particular focus on predicting root water uptake.

We address this problem by first rigorously reducing the coupled 3D Richards equation, governing unsaturated flow in the soil, and the 3D Stokes equation, describing water flow within the root xylem, to a well-posed 3D-1D formulation. Then, we cast the problem within a PDE-constrained optimization framework, enabling a direct and efficient quantification of root water uptake. To handle the non-linearity of the Richards equation and its coupling with the 1D root network, we design a customized iterative solver, balancing accuracy with computational efficiency.

A key aspect of our work is the pioneering application of the Virtual Element Method (VEM) for the spatial discretization of the three-dimensional soil domain. VEM ability to handle general polyhedral meshes, including elements with aligned edges and faces as well as concave elements, significantly enhances the method capacity to address realistic scenarios. This flexibility simplifies, for instance, the meshing of irregularly shaped soil environments, such as complex geometries arising from the presence of impervious obstacles like stones in soil. For the one-dimensional root xylem, we employ a mixed Finite Element formulation, carefully managing the degrees of freedom at branching points through strong flux conservation.

Additionally, we integrate a dynamic root architecture growth model, driven by a discrete-hybrid tip-tracking algorithm that accounts for key plant tropisms such as hydrotropism, geotropism, and exotropism. Notably, we introduce a novel method for modeling negative thigmotropism, i.e. the natural tendency of plants to avoid obstacles, leveraging an auxiliary distance field and the properties of the VEM basis functions. This approach removes the need for computationally expensive iterative schemes during growth simulation.

Several numerical examples are presented, demonstrating the accuracy and scalability of the method, and highlighting its potential for application in large-scale, realistic root–soil interaction studies.

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Loosely coupled schemes for fluid-structure interactions

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Over the past decade, finite element methods and loosely coupled schemes, which decouple fluid-structure interactions (FSI) into separate fluid and solid sub-problems solved sequentially, have been extensively studied for incompressible FSI. These schemes are widely adopted in practice due to their simplicity and low computational cost. Despite their popularity, the development of stable, fully discrete schemes with high-order temporal accuracy remains a significant challenge. Existing stable schemes for coupled FSI models are limited to first-order temporal accuracy. Furthermore, spatially optimal L2-norm error analysis is both technically demanding and rarely explored in the research community. This talk will focus on these two key challenges and present our recent progress in addressing them.

Joint work with W. Sun.

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MS04 – Iterative and direct solvers for optimization and inverse problems

Organizers: Marcella Bonazzoli, Liu-Di Lu, Tommaso Vanzan

Optimization and inverse problems are ubiquitous in applied mathematics, with applications ranging from engineering and physics to imaging and data science. The complexity of these phenomena, such as multiphysics formulation, high frequency wave propagation, and long timescale, often necessitates advance tailored discretizations, which eventually lead to very large systems and require using efficient numerical techniques. Solvers both iterative and direct play a crucial role in addressing these problems. This minisymposium explores recent advances and emerging trends in the development of these solvers, discussing both theoretical insights, numerical implementations and practical applications.

List of Speakers

- M. Bonazzoli (Inria, Institut Polytechnique de Paris) – *Seismic imaging of a dam-rock interface using high performance computing*.
- D. Q. Bui (Inria Saclay) – *Optimized Schwarz methods in time for discrete transport control*.
- V. Dolean (Eindhoven University of Technology) – *Numerical modeling for shoulder injury detection using microwave imaging*.
- F. Kwok (Université Laval) – *Convergence of ParaOpt for Runge-Kutta time discretizations*.
- U. Langer (Johannes Kepler University Linz) – *Robust space-time finite element solvers for distributed hyperbolic optimal control problems*.
- R. Löscher (TU Graz) – *Dirichlet boundary control subject to the Laplace equation with $H^{1/2}(\partial\Omega)$-regularization*.
- L. Lu (University of Geneva) – *Some recent results on time domain decomposition methods for PDE-constrained optimization*.
- B. Mandal (IT Bhubaneswar) – *Optimal control of sub-diffusion problem using substructuring algorithms*.
- C. Milano (University of Reims Champagne Ardenne) – *Numerical method for electromagnetic cartography in medical imaging*.
- T. Nguyen (Max Planck Institute for Solar System Research) – *Bi-level iterative regularization for inverse problems in nonlinear PDEs and applications*.
- A. Sior (University of Liège) – *Numerical investigation of a multi-step one-shot method for frequency domain acoustic full waveform inversion*.
- D. Tognon (INRIA Paris) – *A parallel-in-time algorithm based on ParaExp for optimality systems*.

Seismic imaging of a dam-rock interface using high performance computing

Marcella Bonazzoli

Inria, Institut Polytechnique de Paris

In this talk, we are interested in reconstructing the interface between the concrete structure of a hydroelectric gravity dam and the underlying rock. Indeed, geophysics literature shows that the roughness of the dam-rock interface has an effect on the sliding stability of gravity dams. We solve the shape optimization inverse problem by using a quantitative imaging method called Full Waveform Inversion. It consists of minimizing a regularized misfit cost functional by computing its shape derivative and iteratively updating the interface shape by the gradient descent method. At each iteration, we simulate time-harmonic elasto-acoustic wave propagation models, coupling linear elasticity in the solid medium with acoustics in the reservoir. More precisely, we need to solve a forward problem for each source, a corresponding adjoint problem for each receiver, all for each frequency. The mesh size is constrained not only by the wavelength, but also by the size of the support of sources and receivers, which is much smaller than the mesh size. Therefore, high-performance computing techniques need to be employed. Numerical results using realistic noisy synthetic data demonstrate the method ability to accurately reconstruct the dam-rock interface, also with a limited number of measurements, in a reasonable computing time.

Joint work with L. Audibert, M.A. Boukraa, H. Haddar, and D. Vautrin.

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Optimized Schwarz methods in time for discrete transport control

Duc Quang Bui

Inria Saclay

We investigate optimized Schwarz domain decomposition methods in time for the control of the 1D transport equation. In the case of an internal control over the whole domain, the optimization problem can be transformed into a system of two coupled PDEs. We then apply the time-domain decomposition (without overlap) strategy to this PDE system as well as on its discretized counterpart. Under Fourier analysis, we analyze three different iterations: the fixed-point iteration, the relaxed iteration, and preconditioned GMRES. For each case, we propose parameters for the transmission conditions that lead to fast convergence of the method. We illustrate our results by numerical examples.

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Numerical modeling for shoulder injury detection using microwave imaging

Victorita Dolean

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Rotator cuff tear (RCT) is one of the most common shoulder injuries, which can be irreparable if it develops to a severe condition. A portable imaging system for the on-site detection of RCT is necessary to identify its extent for early diagnosis. We introduce a microwave tomography system, using state-of-the-art numerical modeling and parallel computing for detection of RCT. The results show that the proposed method is capable of accurately detecting and localizing this injury in different size. In the next step, an efficient design in terms of computing time and complexity is proposed to detect the variations in the injured model with respect to the healthy model. The method is based on finite element discretization and uses parallel preconditioners from the domain decomposition method to accelerate computations. It is implemented using the open source FreeFEM software.

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Convergence of ParaOpt for Runge-Kutta time discretizations

Felix Kwok

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The ParaOpt algorithm was introduced in 2019 as a way of parallelizing the solution of first-order optimality systems arising from optimal control problems. The method was inspired by Parareal and can be regarded as an approximate Newton method applied to a multiple shooting problem, where the Jacobian solve is approximated by a coarser discretization in time. Its convergence properties have been analyzed in the original paper for linear-quadratic optimal control problems, where the underlying system is dissipative and discretized in time by the implicit Euler method. In this talk, we will consider the behaviour of the method for linear ODE systems discretized using higher order Runge-Kutta methods: this new result is based on an operator analysis and does not require the ODE system to be dissipative. We show that under some additional assumptions on the Runge-Kutta method, the contraction factor of ParaOpt is proportional to Δt^p, where Δt is the time step of the coarse discretization and p is the order of the Runge-Kutta method. In other words, the higher the order of the Runge-Kutta discretization, the faster ParaOpt converges. Numerical examples illustrate the behaviour of the method.

Joint work with J. Salomon and D.N. Tognon.

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Robust space-time finite element solvers for distributed hyperbolic optimal control problems

Ulrich Langer

Johannes Kepler University Linz

We propose, analyze, and test new robust iterative solvers for systems of linear algebraic equations arising from the space-time finite element (fe) discretization of reduced optimality systems defining the approximate solution of hyperbolic distributed, tracking-type optimal control problems with both the standard L_2 and the more general energy regularizations. In contrast to the usual time-stepping approach, we discretize the optimality system by space-time continuous piecewise-linear fe basis functions which are defined on fully unstructured simplicial meshes. If we aim at the asymptotically best approximation of the given desired state y_d by the computed fe state $y_{\varrho h}$, then the optimal choice of the regularization parameter ϱ is linked to the space-time fe mesh-size h by the relations $\varrho = h^4$ and $\varrho = h^2$ for the L_2 and the energy regularization, respectively. For this setting, we can construct robust (parallel) iterative solvers for the reduced fe optimality systems. In practice, we use these solvers in a nested iteration framework that allows us to control the accuracy of the state iterates and the cost of the control in a systematic way on a sequence of uniformly or adaptively refined meshes. In the case of adaptively refined meshes, we may use variable regularization parameters adapted to the local behavior of the mesh-size. The numerical results illustrate the theoretical findings firmly.

Joint work with R. Löscher, O. Steinbach, and H. Yang.

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Dirichlet boundary control subject to the Laplace equation with $H^{1/2}(\partial\Omega)$-regularization

Richard Löscher

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We briefly recall an abstract framework for the analysis of optimal control problems subject to partial differential equations. We then cast the Dirichlet boundary control into this setting, considering the cost in $H^{1/2}(\partial\Omega)$, which is the natural norm (or energy norm) when considering the state to be in $H^1(\Omega)$. We discuss regularization and finite element error estimates enabling us to derive an optimal relation between the finite element mesh size and the regularization parameter, balancing the cost of the control and the accuracy of the approximation of the desired state. This relationship is crucial in designing efficient solvers. Our theoretical findings are complemented by numerical examples, including examples with discontinuous targets.

Joint work with U. Langer, O. Steinbach, and H. Yang.

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Some recent results on time domain decomposition methods for PDE-constrained optimization

Liu-Di Lu

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PDE-constrained optimization problems arise in a wide range of applications, including aerodynamics, mathematical finance, bioprocesses, and epidemiology. Using the Lagrange multiplier technique, optimal solutions can be characterized by the first-order optimality system. When the governing PDEs are time-dependent, this system typically exhibits a forward-backward structure, and classical time stepping methods cannot be applied to solve this system. Solving the entire system at once can become computationally expensive, especially in higher spatial dimensions. To address this challenge, parallelization techniques are crucial. In this talk, I will present recent developments in time domain decomposition methods for these problems, supported by both theoretical results and numerical examples.

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Optimal Control of sub-diffusion problem using substructuring algorithms

Bankim Mandal

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We explore the convergence patterns of Neumann-Neumann and Dirichlet-Neumann waveform relaxation algorithms when applied to distributed optimal control problems constrained by sub-diffusive and diffusive partial differential equations. For regular one-dimensional domains with multiple subdomains, we investigate how varying diffusion coefficients affect the convegence of these algorithms. Using a semi-discrete approach, we discretize time while maintaining spatial continuity. We use a both-sided graded mesh in time for the analysis that enhances convergence speed compared to traditional mesh grading methods used in time-fractional PDEs. All analytical results are validated through extensive numerical experiments.

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Numerical method for electromagnetic cartography in medical imaging

Charlotte Milano

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Magnetic Resonance based Electric Properties Tomography (MR-EPT) is a non-invasive and non-ionizing technique that estimates the electric properties (permittivity and conductivity) of biological tissues from Magnetic Resonance Imaging (MRI) data. This modality has gained interest since it is able to provide reliable information about pathological and healthy tissues of the human body. Moreover, the precise knowledge of the electric properties is necessary in numerical simulations of MRI to guarantee the international safety standards during the medical exam.

The aim of the present work is the reconstruction of the electric properties of the human brain from measurements of the radiofrequency field generated in a magnetic resonance scanner. As MR-EPT is an inverse parameter problem, it requires a robust numerical method.

We focus here on the "Contrast Source Inversion" method (CSI) which reformulates the original inverse parameter problem as an inverse source problem with two unknowns: the contrast source related to the data operator, and the contrast function related via a state equation to the unknown electric parameters. The minimization of the corresponding cost functional which is the weighted sum of the data and state errors, is performed by an iterative two-step method. In a first step, the contrast function is updated by a conjugate gradient method for a fixed contrast function. The new contrast function is then computed as the minimizer of the state error term.

We consider a two-dimensional transverse magnetic setting for which the electric field formulation of Maxwell's equations reduces to the scalar Helmholtz equation with the contrast function acting as a source term. The data operator then defines the link between the electric field perturbed by the presence of the object and the magnetic field data. The CSI algorithm involves the resolution of three linear systems at each iteration corresponding to the finite element discretization of one forward and two adjoint problems. The matrices of the forward and adjoint problem are the same for all iterations and it is thus advantageous to use a direct solver since assembly and factorization can be performed once and for all during the initialization step of the algorithm. If factorization of the whole matrices would not be possible due to the need of refined meshes, decomposition techniques could be applied to keep the computational cost reasonable.

We present numerical results for a realistic head model and synthetic measurements obtained by simulation of the electromagnetic field in an MRI birdcage coil.

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Bi-level iterative regularization for inverse problems in non-linear PDEs and applications

Tram Nguyen

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In this talk, I present my novel bi-level regularization methods for ill-posed inverse problems governed by nonlinear partial differential equations (PDEs). By a bi-level Landweber scheme, one alternates between upper-level parameter reconstruction and lower-level state approximation. We derive stopping rules for lower- and upper-level iterations, and prove convergence and acceleration of the bi-level algorithm. The bi-level approach illustrates its universality through several reaction-diffusion applications in which the nonlinear reaction law needs to be determined.

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Numerical investigation of a multi-step one-shot method for frequency domain acoustic full waveform inversion

Alejandro Sior

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Full waveform inversion is an imaging method consisting in the spatial reconstruction of the parameter field of a wave propagation problem through the optimization of a misfit functional between the synthetic solution of the problem and the observed solution. Full waveform inversion is typically articulated in two distinct layers: the inner layer consisting in solving the forward and adjoint problems for the calculation of the misfit gradient, and the outer layer performing the optimization.

For large scale problems, the forward and adjoint problems need to be solved using an iterative domain decomposition method. The one-shot paradigm couples these iterative solutions with the optimization steps by limiting the number of forward and adjoint solver iterations to a small number, thus producing an inexact gradient.

In this talk, we investigate the practical feasibility and performance of the one-shot paradigm in the context of full waveform inversion in the frequency domain. We apply the one-shot algorithm proposed in [1] with gradient descent for minimization and an Optimized Restricted Additive Schwartz-preconditioned stationary iterative solver for the forward and adjoint problems. The computational cost is investigated in terms of the number of forward and adjoint iterations, the gradient descent step and the number of subdomains.

Joint work with C. Guezaine and B. Martin.

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A parallel-in-time algorithm based on ParaExp for optimality systems

Djahou Tognon

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In this talk, I will present a new parallel-in-time algorithm for solving optimal control problems constrained by discretized partial differential equations. Our approach which is based on a deeper understanding of ParaExp, considers an overlapping time-domain decomposition in which we combine the solution of homogeneous problems using exponential propagation with the local solutions of inhomogeneous problems. The algorithm yields a linear system whose matrix-vector product can be fully performed in parallel. We will present a preconditioner to speed up the convergence of GMRES in the special cases of the heat and wave equations.

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MS05 – Domain decomposition methods for multiphysics and heterogeneous problems

Organizers: Michele Botti, Ilario Mazzieri

Domain decomposition methods are powerful tools for solving multiphysics and heterogeneous problems by partitioning complex systems into smaller, manageable subdomains. These techniques enable the coupling of different physical models and numerical schemes, facilitating efficient parallel computations and enhanced flexibility in handling heterogeneous materials and interfaces. This minisymposium explores new theoretical results, algorithmic developments, and practical applications of domain decomposition methods, emphasizing their role in tackling challenges heterogeneous problems in engineering, physics, and applied mathematics.

List of Speakers

- F. Bonaldi (Université de Perpignan) – *An energy-consistent discretization of hyper-viscoelastic contact models for soft tissues*.
- F. Cumaru (Delft University of Technology) – *Solving large-scale heterogeneous problems using overlapping Schwarz preconditioners with algebraic multiscale coarse spaces*.
- P. Gervasio (Università degli Studi di Brescia) – *An overlapping coupling framework for Stokes-Darcy equations*.
- J. Manyer (Monash University) – *BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part II: the preconditioner*.
- V. Martin (Universite de Picardie Jules Verne) – *Optimized Schwarz waveform relaxation methods for wave-heat coupling*.
- A. Naegel (Goethe University Frankfurt) – *Scalable multilevel solvers for linear poroelasticity*.
- F. Renzi (Politecnico di Milano) – *Stability of a new loosely coupled fluid-structure interaction scheme in hemodynamics*.
- P. Strohbeck (University of Stuttgart) – *Optimized Schwarz method for the Stokes-Darcy problem with generalized interface conditions*.
- M. Tayachi (Université Grenoble Alpes and Grenoble INP) – *Design and theoretical analysis of a Schwarz coupling method for linearized 3D Navier-Stokes equations and linearized 2D shallow water equations*.
- J. Tushar (Monash University) – *BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part I: a discrete trace theory*.
- T. Vanzan (Politecnico di Torino) – *Optimized Schwarz methods for the time-dependent Stokes-Darcy system*.
- Y. Xu (Northeast Normal University) – *Optimized Schwarz waveform relaxation for heat-viscoelastic structure interaction*.

An energy-consistent discretization of hyper-viscoelastic contact models for soft tissues

Francesco Bonaldi

Université de Perpignan

In this work, we propose a mathematical model of hyper-viscoelastic problems applied to soft biological tissues, along with an energy-consistent numerical approximation. We first present the general problem in a dynamic regime, with certain types of dissipative constitutive assumptions. We then provide a numerical approximation of this problem, with the main objective of respecting energy consistency during contact in adequacy with the continuous framework. Given the presence of friction or viscosity, a dissipation of mechanical energy is expected. Moreover, we are interested in the numerical simulation of the non-smooth and non-linear problem considered, and more particularly in the optimization of Newton's semi-smooth method and Primal Dual Active Set (PDAS) approaches. Finally, we test such numerical schemes on academic and real-life scenarios, the latter representing the contact deployment of a stainless-steel stent in an arterial tissue.

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Solving large-scale heterogeneous problems using overlapping Schwarz preconditioners with algebraic multiscale coarse spaces

Filipe Cumaru

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The two-level overlapping additive Schwarz method provides a robust and scalable preconditioning scheme for linear systems resulting from the discretization of elliptic problems. In this framework, a coarse space is used to solve a global coupling problem. A well-chosen coarse space improves the robustness of the preconditioner by better representing changes in the underlying problem properties, thus guaranteeing scalability for a wider range of problems. In this talk, we consider an algebraic multiscale coarse space. We present a parallel implementation of this coarse space on the FROSch (Fast and Robust Overlapping Schwarz) library, part of the Trilinos software framework. Moreover, we explore algebraic enrichment strategies to extend robustness to larger sets of problems. Finally, we assess the parallel performance and efficiency of the algebraic multiscale coarse space for heterogeneous problems by comparing it against other algebraic coarse space options available on FROSch. We also verify the robustness of our enrichment strategies on different coefficient functions.

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An overlapping coupling framework for Stokes-Darcy equations

Paola Gervasio

Università degli Studi di Brescia

To model fluid filtration through porous media we couple the Stokes and Darcy equations by the Interface Control Domain Decomposition (ICDD) method.

Differently from the commonly used approach based on the Beavers-Joseph-Saffman coupling conditions at a sharp interface, the ICDD method [1,2] considers two overlapping subdomains and it looks for local solutions satisfying minimum jumps for both velocity and pressure on the internal boundaries of the subdomains.

In this talk we validate the ICDD method, by comparing its solution with the one computed by solving the Stokes equations at the microscale in the whole computational domain. Our analysis allowed us to identify the best width of the overlapping region and its position inside the transition zone between the free–fluid and porous-medium regime. Finally, in the case of homogeneous porous media, we show that the ICDD solution is an approximation of order ε of the Stokes solution at the microscale, where ε is the ratio between the micro and the macroscale.

Joint work with M. Discacciati.

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BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part II: the preconditioner.

Jordi Manyer

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Polytopal methods are a class of finite element methods that can be applied on polytopal meshes, i.e, meshes that are composed of general polygons or polyhedra. Polytopal methods have become increasingly popular since they can deal with complex geometries without the need for complex meshing tools. These often involve hybrid spaces, where degrees of freedom are attached to mesh entities of different dimensions.

The design of efficient, robust, scalable solvers for linear systems arising from these kind of discretizations is important to make them competitive with traditional methods on real world applications. One family of such scalable preconditioners are non-overlapping domain decomposition methods.

The analysis of these preconditioners relies on bounds for the so-called trace and lifting operators. For conforming methods, these bounds are based on continuous trace theory and are well understood. This is not the case for non-conforming methods, such as polytopal methods, since the trace of piecewise polynomial functions in $L^2(\Omega)$ does not possess $H^{1/2}(\partial\Omega)$ regularity. A novel discrete trace theory for polytopal methods [1] has been introduced by *Jai Tushar* in the companion talk *"BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part I: A discrete trace theory"*. The theory provides a robust framework to analyse non-overlapping domain decomposition preconditioners for polytopal methods.

In this talk, we will use this theory to prove optimal condition number estimates for the Balancing Domain Decomposition by Constraints (BDDC) preconditioner. We numerically validate our claims for the HDG and HHO schemes, on both structured and fully polytopal meshes, in 2D and 3D. We show robustness and scalability of our preconditioner for up to several hundreds of processors. All experiments are performed using the open-source finite-element library Gridap, and run on the GADI supercomputer using resources provided by the Australian Government through NCI under the NCMAS Merit Allocation Scheme. This talk is based on [2,3].

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Optimized Schwarz waveform relaxation methods for wave-heat coupling

Veronique Martin

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Heterogeneous domain decomposition methods are domain decomposition methods where one does not solve the same equation in each subdomain. One motivation for this is when the physics are different in different subdomains. We focus here on the model problem of coupling a heat and a wave equation in one spatial dimension. This is a minimal example for more sophisticated fluid structure coupling, for which we can perform a complete analysis: we design and analyze optimized transmission conditions in the context of a Schwarz Waveform Relaxation algorithm. We illustrate our results with numerical experiments.

Joint work with F. Chouly and M.J. Gander.

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Scalable multilevel solvers for linear poroelasticity

Arne Naegel

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Developing efficient solvers for coupled PDE systems is often a non-trivial task, since one must combine suitable schemes for time integration and linear solvers, which are suitable for HPC systems. In this study, we suggest a combination of methods for the quasi-static Biot system.

The fixed stress iteration, see e.g. [4], can be interpreted as a special block-LU decomposition for the coupled system. In this case, degrees of freedom for deformations and pressures are separated. It is a key observation that the Schur complement, when formed w.r.t the pressure, can be approximated by a properly scaled identity (e.g., [5]). The method can be generalized to deal with jumping coefficients in heterogeneous media (e.g., [1]).

We avoid the aforementioned splitting and employ a multigrid solver for the fully-coupled system. The method is based on the fixed-stress smoothers suggested by Gaspar und Rodrigo, 2017. Additional acceleration is achieved by a multigrid reduction in time [2]. We investigate robustness and provide a scaling study in an HPC environment. Aspects of the implementation in the software UG4 [6] are discussed.

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Stability of a new loosely coupled fluid-structure interaction scheme in hemodynamics

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The efficient solution of fluid-structure interaction (FSI) problems poses significant challenges due to the strong coupling between the two heterogeneous physics, especially in applications, such as hemodynamics, where fluid and solid densities are comparable leading to a high added mass effect, which can jeopardize the stability and convergence properties of the numerical scheme. Among the existing strategies, loosely coupled (LC) partitioned approaches, which are based on the solution of one fluid and one structure problem at each time step, offer an efficient alternative to strongly coupled (SC) partitioned procedures and monolithic schemes. Indeed, LC solvers allow for modular treatment of the two subproblems, unlike monolithic solvers, and may reduce computational cost with respect to the SC partitioned procedures. However, their stability properties deteriorate in regimes where fluid and solid densities are comparable, such as in cardiovascular simulations [1]. In this context, our aim is to present a new coupling strategy to obtain a stable LC scheme in the presence of high added mass. Starting from the interpretation of the standard Dirichlet-Neumann (DN) scheme as a Richardson method with a block Gauss-Seidel preconditioner and acceleration parameter $\alpha = 1$, we consider the SC partitioned method associated with an arbitrary value of α, which leads to a new DN-like scheme (DN-α algorithm), where correction terms appear that could improve the convergence of the standard DN method. We perform a convergence analysis of the DN-α scheme over the toy problem proposed in [1], showing convergence for a specific range of α without the need for any relaxation, without the standard DN method. Secondly, building on the DN-α scheme, we propose a new LC scheme (DN-α-LC) obtained by performing just one iteration per time step (corresponding to one iteration of the Richardson method). We provide a stability analysis over the same toy problem above, proving that the proposed LC scheme is conditionally stable under a constraint on the time step and the Richardson parameter α, with stability depending on the ratio between fluid and solid densities. All the theoretical findings of this work are supported by numerical experiments in the hemodynamics regime. In particular, these results show that the solution of the DN-α-LC scheme converges to the DN-α one as the time step is reduced. Such results confirm the effectiveness and applicability of the proposed schemes in cardiovascular simulations.

Joint work with C. Vergara.

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Optimized Schwarz method for the Stokes-Darcy problem with generalized interface conditions

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In this talk, we present a coupled Stokes-Darcy problem where the local subproblems are defined in non-overlapping subdomains, and the coupling conditions involve a generalization of the classical Beavers-Joseph interface condition. The generalized condition was obtained using homogenization and boundary layer theory, and, differently from the Beavers-Joseph condition, it is applicable when the fluid flow is arbitrarily directed to the fluid-porous interface.

To solve the Stokes-Darcy problem with this generalized interface conditions efficiently, we design a Robin-Robin domain decomposition method, and we use Fourier analysis to identify optimal weights in the Robin interface conditions. We study efficiency and robustness of the proposed method, and we present numerical simulations which confirm our theoretical results.

Joint work with M. Discacciati and I. Rybak

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Design and theoretical analysis of a Schwarz coupling method for linearized 3D Navier-Stokes equations and linearized 2D shallow water equations

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When dealing with the simulation of complex physical phenomena and in order to avoid heavy numerical simulations, one can reduce a complex model in some locations and replace it, if the physics allows it, by the simplest ones usually obtained after simplifications. Such simplifications in the model may involve a change in the geometry and the dimension of the physical domain. In that case, one deals with a dimensionally heterogeneous coupling problem. We consider in this talk the case of 3D linearized hydrostatic Navier-Stokes equations coupled with corresponding 2D linearized shallow water equations. We will show briefly how to derive the 2D linearized shallow water system from the 3D linearized hydrostatic Navier-Stokes system. Then, a Schwarz-like algorithm to couple the two systems is proposed and studied. We will show that under some assumptions, the convergence of this Schwarz algorithm is equivalent to the convergence of the classical domain decomposition algorithm of shallow water equations. Finally, some theoretical results related to the control of the difference between the global 3D reference solution and the 3D part of the coupled solution are given. These results are illustrated numerically.

Joint work with C. Acary-Robert and E. Blayo.

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BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part I: a discrete trace theory.

Jai Tushar

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The analysis of non-overlapping domain decomposition method-based solvers like BDDC relies on the exchange of information across intersubdomain boundaries. It requires three main ingredients: a *trace inequality*, which implies that the restriction of functions to the subdomain interface is stable; a *lifting result*, which lifts this restriction to the interior of the neighboring subdomain; and continuity of a *face truncation operator* on piecewise polynomial functions. The bound on this operator leads to a mesh-dependent logarithmic estimate.

For conforming finite element methods, this is realized with the help of continuous trace theory. For non-conforming methods, such as polytopal methods, the continuous trace theory fails, since the trace of piecewise polynomial functions in $L^2(\Omega)$ does not possess $H^{1/2}(\partial\Omega)$ regularity. The current state of the art to address this involves constructing an interpolant of a function on the interface (intersubdomain boundary) onto a conforming finite element space and then applying the continuous trace theory. As a result, all the analysis for non-conforming spaces so far has been carried out on conforming simplicial/tetrahedral or quadrilateral/hexahedral meshes.

In this talk, we will present a discrete trace theory for non-conforming polytopal methods. This theory is based entirely on the fully discrete hybrid spaces appearing in these methods. It hinges on the design of a novel discrete trace seminorm. For this seminorm, we establish discrete trace and lifting inequalities that are independent of mesh size and hold on quasi-uniform polytopal meshes. We also derive a truncation estimate in this discrete trace seminorm for piecewise polynomials in a hybrid setting. Finally, we compute the proposed discrete trace operator and show that its spectrum is equivalent to that of the discrete energy operator (a consequence of the discrete trace and lifting inequalities). This talk is based on [1].

The tools presented in this talk will be used in the talk by *Jordi Manyer* titled *"BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part II: The preconditioner"* to prove the condition number bounds of the BDDC preconditioner for non-conforming polytopal methods [2].

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Optimized Schwarz methods for the time-dependent Stokes-Darcy system

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The Stokes–Darcy system has been extensively studied during the last two decades due to its relevance to model filtration phenomena in industrial and natural applications. In this talk, we first recall a few works on optimized Schwarz methods for the stationary problem, highlighting the theoretical challenges in deriving optimized transmission conditions, but also their strong effectiveness. We then delve into the time-dependent coupled system, discretized with a general theta method, and derive the optimized transmission conditions that depend on both the physical and discretization parameters. The analysis leads to a particularly challenging min-max problem, which is successfully tackled with a novel approach. Finally, numerical experiments do confirm the effectiveness of the iterative scheme, even for challenging realistic ranges of the physical parameters.

Joint work with M. Discacciati.

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Optimized Schwarz waveform relaxation for heat-viscoelastic structure interaction

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The Optimized Schwarz Waveform Relaxation (OSWR) method decomposes the heat-viscoelastic interaction model into two single physical processes, solved using existing numerical methods and coupled via transmission conditions to reconstruct system behavior. Efficient transmission conditions are critical for rapid convergence. In this study, we propose three Robin-type transmission conditions by relaxing interface coupling. For the standard Robin condition, convergence is rigorously proven via energy estimate. Fourier analysis further characterizes the convergence factor in the frequency domain, enabling optimization of relaxation parameters through asymptotic analysis and high-frequency approximations. Our results reveal, for the first time in multi-physics systems, that "heterogeneity" (thermal conductivity and damping coefficient contrast) significantly impacts OSWR's performance. Specifically, higher "heterogeneity" accelerates convergence for both scaled and two-sided Robin conditions. Notably, the two-sided Robin OSWR is mesh-independent, depending solely on "heterogeneity". Numerical experiments validate these theoretical findings.

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MS06 – Domain decomposition based methods for cardiovascular simulation

Organizers: Michele Bucelli, Francesca Renzi, Christian Vergara, Alfio Quarteroni

The aim of this mini-symposium is to collect original contributions in the field of Domain Decomposition (DD) methods applied to the cardiovascular system. This comprises the description of vascular hemodynamics with possible heterogeneous coupling coming from the geometric multiscale approach and fluid-structure interaction (FSI), as well as the modeling of the cardiac function. For the latter, there is the need of accounting for the coupling among different physics, such as the propagation of the electrical impulse through the cardiac tissue, the myocardial tissue's mechanical contraction and relaxation processes, blood dynamics in the cardiac chambers, and immersed cardiac valves dynamics, resulting in electro-mechanics and FSI (or even EFSI) couplings. Possibly, the vascular (even by means of a lumped parameters model) and the cardiac systems could be coupled themselves to have an overall picture of the cardiovascular system. The numerical discretization of the equations underlying such interdependent events demands the stable, accurate and efficient integration of different numerical methods, suitably coupled in a DD framework, which represents an effective approach for tackling the multi-physical nature of the cardiovascular system. In such a context, the development and application of appropriate robust solvers is mandatory to achieve significant results in a reasonable computational time, which is often a key requirement for clinical applications.

A further aim of this mini-symposium is to offer a valuable opportunity for academics and professionals to actively share their latest findings, exchange insights, and establish connections that will inspire future collaborations, accelerate research, and foster translational activities in this dynamic field.

List of Speakers

- E. Centofanti (University of Pavia) – *Scalable and multilevel preconditioners for composite discontinuous Galerkin discretizations of the cardiac EMI model*.
- A. Hasaballa (University of Oxford) – *Multiscale modelling of hypertrophic cardiomyopathy progression and targeted therapy development*.
- R. Krause (KAUST) – *Non-conforming coupling in cardiac simulations*.
- G. Montino Pelagi (LaBS, DCMC, Politecnico di Milano) – *A multiphysics coupled model for the entire coronary circulation: methods, calibration and applications*.
- T. Simpson (Università della Svizzera italiana) – *Computational tools for cardiac simulation - GPU parallel matrix-free mesh-free multigrid*.
- R. Tenderini (EPFL) – *Model order reduction of hemodynamics by space-time reduced basis and reduced fluid-structure interaction*.
- A. Veneziani (Department of Mathematics, Department of Computer Science, Emory University) – *A domain-decomposition framework for cardiac radiofrequency ablation: implementation and analysis*.
- T. Wick (Leibniz University Hannover) – *A sustainable and accessible restricted Schwarz preconditioner by coupling deal.II and FROSCH applied to fluid-structure interaction*.

Scalable and multilevel preconditioners for composite discontinuous Galerkin discretizations of the cardiac EMI model

Edoardo Centofanti

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The EMI (Extracellular space, cell Membrane, and Intracellular space) model provides a microscale framework for simulating cardiac electrical activity, capturing natural discontinuities across domain interfaces and the spatiotemporal evolution of ionic charge distributions on cell membranes. In this work, we investigate scalable preconditioning strategies for the composite Discontinuous Galerkin (DG) discretization of the EMI model, focusing on the generalized Dryja-Smith-Widlund (GDSW) preconditioner and algebraic multigrid (AMG) methods. The GDSW preconditioner is shown to ensure scalability and quasi-optimal convergence, convergence, confirming the theoretical bounds on the condition number, while taylored EMI-AMG preconditioners leverage contemporary computational power for large-scale systems. Numerical experiments demonstrate the robustness of the preconditioners also under ischemic conditions of the in-silico model. Performance evaluations on hybrid CPU-GPU architectures are also discussed, with a comparative analysis of the scalability properties of the two preconditioners, providing critical insights into the effectiveness of multilevel solvers for the EMI model.

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Multiscale modelling of hypertrophic cardiomyopathy progression and targeted therapy development

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Hypertrophic cardiomyopathy (HCM) is a genetically inherited cardiac disorder affecting at least 1 in 500 individuals and remains a leading cause of sudden cardiac death in young adults. Despite advances in genetic diagnostics, clinical management remains largely symptom-oriented due to the complex and heterogeneous nature of HCM and limited mechanistic insight into its progression. We present a human-based multiscale simulation framework that bridges subcellular ionic processes to whole-organ cardiac function through a tightly coupled, multiphysics formulation. Developed using Alya—a high-performance, domain-decomposed finite element code optimised for supercomputers—the framework integrates cellular electrophysiology and myocardial mechanics within a unified electromechanical model. Electrical activation is governed by the monodomain equation with orthotropic conduction, incorporating the ToR-ORd ionic model to capture detailed human ventricular action potential kinetics. These subcellular signals drive active stress generation via calcium transients through the Land model of excitation–contraction coupling, which in turn deform cardiac tissue represented by a total Lagrangian formulation with fibre-aligned passive mechanics. The full system of nonlinear partial differential equations is solved monolithically on a shared finite element mesh using a domain decomposition strategy, enabling scalable and efficient resolution from the cellular to the organ level across anatomically realistic geometries. Using a healthy biventricular model as a baseline, we simulate three progressive stages of HCM: (I) a non-hypertrophic phenotype driven by sarcomeric gene mutations (e.g. MYH7), (II) the classic hypertrophic phenotype, and (III) advanced remodelling incorporating ionic and structural alterations. Results show that even isolated mutations can lead to elevated contractility and impaired relaxation before the onset of anatomical hypertrophy—mirroring early-stage, subclinical disease. As the disease progresses, diastolic dysfunction and disrupted pressure–volume dynamics emerge, leading to impaired cardiac performance. We further evaluate the dose-dependent effects of Mavacamten, a novel myosin inhibitor. Simulations indicate that hypercontractility is reversible in early disease, but impaired relaxation persists in advanced stages, highlighting the need for stage-specific therapies. This work demonstrates the potential of multiscale, multiphysics in-silico models to mechanistically explore genotype-to-phenotype transitions and to support the development of precision therapies for inherited cardiomyopathies.

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Non-conforming coupling in cardiac simulations

Rolf Krause

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We discuss different "use cases" for decomposition approaches based on variational transfer (L2-projection) for cardiac simulations. More precisely, we will present a non-overlapping approach for the solution of the monodomain equation by means of space-time methods as well as a fully overlapping approach for fluid-structure interaction with contact for bio-prosthetic heart valves. We will show in which way variational transfer allows for designing flexible and massively parallel solution for both problem classes and will illustrate our findings with numerical examples.

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A multiphysics coupled model for the entire coronary circulation: methods, calibration, and applications

Giovanni Montino Pelagi

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Effective modeling of myocardial perfusion would greatly support the non-invasive diagnosis of ischemic heart disease while also providing personalized treatment options. However, there are several modeling challenges to address, including: the multiscale nature of the coronary circulation, the effects of cardiac contraction, a high inter-patient variability in the anatomy and the effects a wide variety of pathological scenarios. We present a multiphysics model of the entire coronary circulation, from the large coronary arteries to the microcirculation, where a 3D fluid-dynamics description of blood flow in the epicardial coronaries is coupled with a multicompartment, compliant Darcy formulation for blood perfusion in the 3D myocardium. Coupling conditions are prescribed at the interfaces to ensure mass conservation and balance of the interface forces, and the coupled problem is solved through a partitioned algorithm with a fixed-point iterative scheme. We also propose a specific way to handle the geometric coupling, representing how blood flow from the large coronaries is distributed in the myocardium, which also accounts for the presence of transverse vessels that are beyond the resolution limit of CT imaging. Perfusion simulations are run on CT-segmented geometries using the high performance Finite Element library lifex (developed at MOX, in cooperation with LaBS, Politecnico di Milano). Model validation is performed at multiple levels by comparing numerical results with invasive measures regarding coronary blood velocity, temporal evolution of the flow profile, pressure drop along the full tree and perfusion distribution at the tissue level. Geometric association between epicardial branches and myocardial mass is validated against a detailed dataset of the entire coronary circulation down to the arterioles (d = 30 μm). We finally introduce two clinical applications of the model, in partnership with Monzino Cardiology Centre (Milan), alongside the strategies that we propose for model calibration in the two cases. In the diagnostic application, we use basic patients measures to build personalized boundary conditions and we calibrate the distal resistances of each coronary branch. The aim is to identify perfusion defects and the numerical results are directly compared with clinically obtained CT perfusion imaging. In the therapeutic application, we also include a calibration of the microcirculation parameters and we employ geometrical alteration of the epicardial arteries to virtually simulate revascularization, so that the optimal treatment can be identified. Validation is done by reproducing the revascularization actually performed on 6 patients and comparing the numerical results with CT perfusion imaging at follow-up.

Joint work with A. Baggiano, G. Pontone, G. Valbusa, and C. Vergara.

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Computational tools for cardiac simulation - GPU parallel matrix-free mesh-free multigrid

Toby Simpson

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Computational science can potentially improve outcomes for millions of people affected by cardiovascular disease. Computer simulation allows low-risk, low-cost design and testing for surgical and pharmacological therapies, as well as tools for automated screening and diagnosis.

The simulation of a human heartbeat requires the solution of a set of physical problems including electrophysiology, elastodynamics, fluid dynamics and fluid-structure interaction, on a range of scales. Current research provides computational models with ever-increasing complexity and computational cost, often requiring hours of processing on supercomputer systems, limiting their application in the clinical setting.

We present a simplified approach that can compute a simulated heart beat on a desktop machine within minutes. We couple the monodomain equation for electrophysiology with finite strain elastodynamics, the Navier Stokes equations for fluid dynamics and the arbitrary Lagrangian-Eulerian method for fluid-structure interaction.

Discretisation is via the finite volume method, using a deforming voxel mesh with geometry encoded by a signed distance function, ideally suited to medical imaging data. Operator splitting with a Helmholtz decomposition gives semi-implicit time integration for fluid and structure.

Central to this formulation, which includes four separate Laplacian problems, is a GPU matrix-free and mesh-free multigrid solver. For a single precision Poisson problem, the solver improves on CPU LAPACK CG by up to three orders of magnitude both in time to solution and memory footprint.

The algorithm is implemented as a set of GPU kernels in the OpenCL standard with no dependence on CPU processing or third-party software.

While the method is readily extensible into other fields of research, we hope that it will allow for closer integration between computational and clinical research in cardiology.

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Model order reduction of hemodynamics by space-time reduced basis and reduced fluid-structure interaction

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In this work, we present a space-time Galerkin reduced basis (ST-GRB) method for the efficient numerical simulation of arterial hemodynamics. By combining the traditional reduced basis (RB) approach with a data-driven low-dimensional compression of temporal dynamics, ST-GRB achieves significant computational gains while preserving accuracy. Compared to existing work on the topic, this study features advances in two key directions. First, we adopt the Navier-Stokes equations to model blood flow, thereby capturing convective effects. To tackle the resulting nonlinearities effectively, we employ a hyper-reduction technique based on approximate space-time affine decompositions. Second, we move beyond the assumption of rigid vessel walls by incorporating wall elasticity directly into the fluid equations by the Coupled Momentum model. A notable feature in this regard is the strategy we propose for the spatio-temporal projection of displacement, which naturally emerges as a by-product of the formulation. Numerical experiments on three-dimensional problems demonstrate that ST-GRB can outperform classical RB approaches, offering accurate approximations at reduced computational costs. However, the method's performance deteriorates when highly complex dynamics or extremely precise solution approximations require a large number of temporal modes, relative to the high-fidelity complexity in time. In the final part of the talk, we also explore how ST-GRB can empower deep learning models by providing efficient, physics-based surrogate PDE solvers for time-dependent regimes.

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A domain-decomposition framework for cardiac radiofrequency ablation: implementation and analysis

Alessandro Veneziani

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Cardiac radiofrequency ablation (RFA) is a possible treatment for arrhythmias. It uses, in general, heat energy to create tiny scars in the heart. The scars are intended to block faulty heart signals and restore a typical heartbeat (see, e.g., mayoclinic.org). A catheter reaches the heart percutaneously. Sensors on the tip of the catheter send electrical signals and record the heart's electrical activity. This information helps to find the area that is causing the irregular heartbeats, and, ultimately, where to apply the treatment. Heat (radiofrequency energy) is then applied to the tissue to block the irregular rhythms. The actual results of the procedure may suffer from a limited understanding of lesion induced, hindering optimal outcomes. Current RFA models often oversimplify the complex multiphysics interactions within heterogeneous domains, neglecting crucial factors.

In this talk we introduce a high-fidelity, multiphysics, and multi-domain computational framework designed to enhance RFA treatment and minimize complications. The framework integrates heat transfer, electrostatics, fluid dynamics, and a three-state cell-death model across electrode, fluid, and tissue regions. Some processes are confined to specific compartments (e.g., fluid dynamics in the fluid, cell-death in the tissue), while others (e.g., electrostatics, heat transfer) span multiple domains. To achieve accurate and scalable simulations, we employ domain-decomposition (DD) approaches. Specifically, Dirichlet-Neumann and Optimized Schwarz-like DD methods are explored and analyzed. The numerical solver is implemented using high-order numerical methods within the MFEM library, and enhanced by efficient partially assembled operators and ongoing GPU acceleration. With this framework, we achieve significant computational efficiency.

The framework is designed to be a versatile platform for RF simulations across diverse tissue types (e.g., kidney, uterine, hepatic) and energy sources (RF, microwave, ultrasound, laser), as well as a range of other biomedical modeling applications (e.g., degradation of bio-resorbable stents).

By advancing computational modeling and data assimilation, this research aims to bridge the gap between theoretical simulations and clinical practice, facilitating the development of more effective, personalized, and safer ablation therapies.

Joint work with E. Cherry, F. Fenton, A. Gizzi, and L. Molinari.

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A sustainable and accessible restricted Schwarz preconditioner by coupling deal.II and FROSCH applied to fluid-structure interaction

Thomas Wick

Leibniz University Hannover

In this presentation, restricted additive Schwarz (RAS) preconditioners from the Trilinos package FROSch (Fast and Robust Overlapping Schwarz) are employed to solve fluid-structure interaction implemented using deal.II (differential equations analysis library). Fluid-structure interaction is modeled with the help of the well-known arbitrary Lagrangian-Eulerian (ALE) approach. Furthermore, a variational-monolithic numerical model is employed. The nonlinear solution is based on Newton's method. Therein, the linear equation systems are solved with GMRES (generalized minimal residuals) and preconditioned with the previously mentioned RAS method. Several numerical experiments in two and three spatial dimensions confirm the performance of the preconditioners as well as the FROSch-deal.II interface. Moreover, cardiovascular configurations such as blood flow simulations in arteries with elastic walls as well as thin flaps are implemented, too.

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MS07 – Efficient parallel solvers for non-Hermitian and indefinite problems

Organizers: Victorita Dolean, Vandana Dwarka, Pierre Marchand, Nicole Spillane

This mini-symposium focuses on recent advances in domain decomposition methods and preconditioning strategies for indefinite and non-symmetric problems. Key topics include the Generalized Optimized Schwarz Method (GOSM), which provides guaranteed fast convergence, even in the presence of cross-points, and its integration with Finite Element Method - Boundary Element Method (FEM-BEM) coupling. Another focus is on coarse spaces for two-level preconditioners where local generalized eigenproblems play a critical role in constructing efficient coarse representations, leading to improved convergence rates. Complementing this, spectral coarsening strategies for non-Hermitian and indefinite systems are explored, offering robust strategies to tackle challenges posed by ill-conditioning. Preconditioning strategies for Helmholtz problems in resonant regimes will also be discussed, addressing the difficulties posed by highly oscillatory solutions. Additional contributions will further expand on domain decomposition approaches for indefinite and non-symmetric problems, providing novel insights into their mathematical foundations and computational implementation.

List of Speakers

- A. Boisneault (CNRS, Inria) – *Generalized optimized Schwarz method, part 2: FEM-BEM coupling*.
- J. Chen (TU Delft) – *An efficient parallel multilevel deflation method for large-scale Helmholtz problems*.
- X. Claeys (UMA, POems, ENSTA, Paris) – *Generalized optimized Schwarz method, part 1: coupling with physical boundaries*.
- V. Dwarka (UMA, Delft University of Technology) – *A convergent classical multigrid method for the 2D Helmholtz equation with high wavenumbers*.
- E. Fressart (Thales Research and Technology and CMAP, CNRS, Ecole polytechnique, Institut Polytechnique de Paris) – *Preconditioners for harmonic Maxwell and quantum linear system solvers*.
- E. Parolin (Inria) – *Coarse spaces for non-Hermitian two-level preconditioners based on local extended generalized eigenproblems*.
- T. Raynaud (POEMS, CNRS, INRIA, ENSTA) – *Preconditioning of GMRES for Helmholtz problems with (quasi-)resonances*.
- R. Scheichl (University of Heidelberg) – *Robust and efficient spectral coarsening for non-Hermitian and indefinite problems*.

Generalized optimized Schwarz method, part 2: FEM-BEM coupling

Antonin Boisneault

POEMS, CNRS, Inria, ENSTA Paris

This talk directly follows the one given by Xavier Claeys in MS07 "Efficient parallel solvers for non-Hermitian and indefinite problems", in which a Generalised Optimised Schwarz Method (GOSM) is introduced for the Helmholtz equation on bounded domains, with weakly imposed classical boundary conditions. It differs from other OSMs by the use of a non-local exchange operator instead of the usual swap operator. This allows us to accommodate the presence of cross-points, that is, points where the interfaces of at least three subdomains intersect.

We present how to extend this work to solve the Helmholtz equation in a complex medium composed of a bounded heterogeneous part and an unbounded homogeneous part. Such problems can be equivalently written using FEM-BEM coupling techniques, which mainly consist in rewriting the problem set in the homogeneous subdomain thanks to Boundary Integral Equations. Then, the FEM-BEM variational formulation of the problem is composed of a volume bilinear form on the heterogeneous subdomain on one hand, and a surface bilinear form on the interface between the two subdomains on the other hand.

Thus, we try to extend the GOSM by replacing the classical boundary conditions with interface conditions arising from several FEM-BEM coupling techniques. We show that only the symmetric Costabel coupling satisfies a key assumption of the GOSM, related to the physical damping property of the system.

We are then able to prove that the GOSM for the Costabel FEM-BEM coupling is geometrically convergent for iterative procedures such that GMRes or Richardson. We present extensive numerical experiments to illustrate the method convergence, even for coupling techniques that do not satisfy the energy loss assumption, and discuss the impact of the choice of transmission operators. We illustrate the fast convergence of the method when non-local transmission operators are considered.

Joint work with M. Bonazzoli, X. Claeys, and P. Marchand.

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An efficient parallel multilevel deflation method for large-scale Helmholtz problems

Jinqiang Chen

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We present a robust and scalable parallel solver for large-scale Helmholtz problems arising in seismic wave modeling. We introduce an advanced multilevel deflation method with a matrix-free implementation that employs re-discretized schemes to approximate the Galerkin coarsening operator, achieving close-to wavenumber-independent convergence. The solver implements a hybrid MPI+OpenMP parallelization framework that optimizes both computational efficiency and memory utilization, enabling the solution of unprecedented problem sizes. The method demonstrates excellent parallel scalability up to thousands of CPU cores while maintaining consistent convergence rates across varying frequencies and problem sizes. The solver's robust performance across diverse geological settings and its excellent scaling properties establish it as a powerful forward modeling engine.

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Generalized optimized Schwarz method, part 1: coupling with physical boundaries

Xavier Claeys

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In this work, we consider a wave propagation in harmonic regime modelled by the Helmholtz equation. We discuss a solution strategy based on the generalized optimized Schwarz method introduced in [1], which leads to a reformulation of the problem as an equation posed on the skeleton of the subdomain partition. In this approach, the wave equation is imposed separately in each subdomain, and transmission conditions coupling subdomains are imposed by means of a so-called exchange operator that can be non-local.

In the present talk, we are specifically interested in the Helmholtz equation posed in a bounded cavity with non-penetrable boundary conditions (e.g. Neumann or Dirichlet boundary conditions), a situation that allows resonance phenomena. We shall discuss the spectral correspondence between the operator of the skeleton formulation and the operator of the original Helmholtz boundary value problem. In particular, in the case where the Helmholtz boundary value problem admits a unique solution, the skeleton reformulation is strongly coercive. If, on the other hand, a resonance phenomenon occurs and the Helmholtz boundary value operator admits non-trivial kernel, we show that the operator of the skeleton formulation admits a non-trivial kernel of the same dimension.

Joint work with A. Boisneault, M. Bonazzoli and P. Marchand.

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A convergent classical multigrid method for the 2D Helmholtz equation with high wavenumbers

Vandana Dwarka

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We introduce the first classical multigrid scheme capable of efficiently solving the highly indefinite two-dimensional Helmholtz equation with wavenumbers up to $k = 500$. Our method supports both constant and variable wavenumber problems and enables fully recursive V- and W-cycles without requiring level-dependent modifications. The method uses standard weighted Jacobi smoothing. A central innovation is the design of higher-order inter-grid transfer operators, which are critical for stability and performance. When paired with a coarse-grid correction based on the Complex Shifted Laplacian (CSL) rather than the original Helmholtz operator, the solver is able to reach convergence and scales linearly with the wavenumber. Additionally, we demonstrate that by employing GMRES(3) smoothing and retaining the higher-order transfer operators, it is possible to achieve convergence rates independent of the wavenumber.

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Preconditioners for harmonic Maxwell and quantum linear system solvers

Elise Fressart

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We consider the solution of large scale time-harmonic Maxwell equations. To this day this problem remains very difficult. Two sources of difficulty are that the equations are neither Hermitian nor semi-definite.

Our approach is to compare different preconditioners both analytically and numerically. Particular emphasis will be on preconditioners that rely on the nearby positive Maxwell equations, for which the Hiptmair-Xu preconditioner is known to be very efficient.

The particularity of our project is that our final objective is to propose a method for solving these equations on quantum computers. We will present some of the difficulties inherent to simulating PDEs on quantum computers as well as some ideas behind state of the art quantum linear system solvers. The efficiency of these solvers critically depends on the condition number of the matrix. This justifies the need for adequate preconditioners.

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Coarse spaces for non-Hermitian two-level preconditioners based on local extended generalized eigenproblems

Emile Parolin

Inria, Paris, France

The scalability of domain decomposition methods relies heavily on the design of the coarse space in a two-level approach. We present a versatile method for constructing coarse spaces for sparse matrices derived from standard PDE discretizations. This method ensures theoretical convergence for fixed point iterative schemes and is applicable to a wide range of problems, including non-Hermitian and indefinite systems, Hermitian preconditioners like additive Schwarz (AS), and non-Hermitian preconditioners such as restricted additive Schwarz (RAS). It also accommodates both exact and inexact subdomain solvers. Our approach involves solving extended generalized eigenproblems locally within each subdomain and applying a carefully chosen operator to the selected eigenvectors to obtain a local discrete solution. This solution is then multiplied by a partition of unity function and extended by zero to form the global coarse space.

Joint work with F. Nataf and P.-H. Tournier.

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Preconditioning of GMRES for Helmholtz problems with (quasi-)resonances

Timothée Raynaud

POEMS (CNRS, INRIA, ENSTA)

Finite element methods are effective for Helmholtz problems involving complex geometries and heterogeneous media. However, the resulting linear systems are often large, indefinite, and challenging for iterative solvers—particularly at high wavenumbers or near resonant conditions.

We derive a GMRES convergence bound that incorporates the nonlinear behavior of the relative residual and relates convergence to harmonic Ritz values. This perspective reveals how small eigenvalues associated with resonances or quasi-resonances can hinder convergence, and when they cease to have an effect.

We illustrate these phenomena through numerical experiments and investigate deflation techniques using (approximate) eigenvectors tailored to resonant regimes. Their impact on GMRES performance is evaluated, including combinations with acceleration methods such as Complex Shifted Laplace preconditioning and domain decomposition.

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Robust and efficient spectral coarsening for non-Hermitian and indefinite problems

Robert Scheichl

University of Heidelberg

Multiscale-Spectral Generalised FE Methods (MS-GFEM) are a powerful tool for approximating solutions to general variational problems that satisfy a Garding-type inequality, including strongly non-Hermitian and highly indefinite problems. They allow to fully localise the construction of optimal approximation spaces via suitable eigenproblems and without any a priori regularity assumptions on the solution or the material parameters. The global approximation error is fully controlled by the local errors which can be rigorously shown to decal nearly exponentially. As an example of its application to indefinite problems, I will consider incompressible Stokes problems with highly heterogeneous or rough viscosity fields, which arise frequently in geophysical and industrial applications. Efficient solution of the resulting saddle point systems remains challenging, particularly in the presence of high viscosity contrasts. A common strategy is to use a Schur complement preconditioner, which isolates the pressure block for separate treatment. However, as the viscosity contrast increases, the approximation of the Schur complement becomes increasingly difficult. To address this challenge, we consider an augmented Lagrangian formulation that modifies the original system by adding a grad-div stabilization term. This shifts the numerical difficulty from the Schur complement to the (elliptic) velocity block with a large near-kernel. We use MS-GFEM to approximate this operator efficiently and use this within the augmented Lagrangian structure as the coarse space in a two-level restricted additive Schwarz preconditioner. The localized basis functions in MS-GFEM are designed to match the changes made by the augmented Lagrangian formulation, so they can effectively capture the rough viscosity field in the modified system. We present theoretical and numerical results demonstrating the robustness and efficiency of the proposed preconditioning strategy across a range of viscosity contrasts involving complex heterogeneous media.

Joint work with C. Alber.

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MS08 – Advanced space-time discretization methods: theory, solvers and applications

Organizers: Ilario Mazzieri, Giancarlo Sangalli, Andrea Moiola

This mini-symposium focuses on the latest advancements in space-time discretization methods for time-dependent PDEs with a special emphasis on domain decomposition solvers. The latter enables efficient parallelization, scalability, and local adaptivity, addressing challenges in large-scale and complex simulations. Topics include theoretical and algorithmic developments and applications in different scientific fields like fluid dynamics, structural mechanics, and multiphysics. The session aims to foster discussions on leveraging space-time approaches and domain decomposition for cutting-edge scientific challenges.

List of Speakers

- W. Boschieri (CNRS, France) – *Fully discrete one-step discontinuous Galerkin schemes on polygonal meshes*.
- T. Elguedj (INSA-Lyon, LaMCoS) – *Fast space-time isogeometric solvers for nonlinear transient heat transfer problems*.
- S. Gomez (University of Milano-Bicocca) – *Robust space-time DG methods for the incompressible Navier-Stokes equations*.
- M. Kern (INRIA) – *A space-time multiscale mortar mixed finite element method for parabolic equations*.
- S. Krell (University Cote d'Azur) – *Classical and optimized overlapping Schwarz methods within the finite volume framework*.
- R. Löscher (TU Graz) – *Space-time finite element methods for the wave equation*.
- M. Mally (TU Darmstadt and University of Santiago de Compostela) – *Parareal and IETI-DP for parabolic PDEs with vanishing time derivatives on certain subdomains*.
- N. Margenberg (Helmut Schmidt University) – *An hp multigrid approach for tensor-product space-time finite element discretizations*.
- I. Perugia (University of Vienna) – *Space-time approximation of the wave equation with splines in time*.
- G. Sangalli (Università di Pavia) – *Isogeometric analysis in space-time*.
- T. Wick (Leibniz University Hannover) – *Space-time single-goal and multi-goal a posteriori error control for adaptively balancing discretization and numerical solver errors*.
- M. Zank (University of Vienna) – *Conforming space-time methods for time-dependent Schrödinger equations*.

Fully discrete one-step discontinuous Galerkin schemes on polygonal meshes

Walter Boschieri

CNRS, France

We propose a new high-order accurate nodal discontinuous Galerkin (DG) method for the solution of nonlinear hyperbolic systems of partial differential equations (PDE) on unstructured polygonal Voronoi meshes. Rather than using classical polynomials of degree N inside each element, in our new approach, the discrete solution is represented by piecewise continuous polynomials of degree N within each Voronoi element, using a continuous finite element basis defined on a subgrid inside each polygon. We call the resulting subgrid basis an agglomerated finite element (AFE) basis for the DG method on general polygons, since it is obtained by the agglomeration of the finite element basis functions associated with the subgrid triangles. The basis functions on each sub-triangle are defined, as usual, on a universal reference element, hence allowing to compute universal mass, flux and stiffness matrices for the subgrid triangles once and for all in a pre-processing stage for the reference element only. Consequently, the construction of an efficient quadrature-free algorithm is possible, despite the unstructured nature of the computational grid.

We also introduce a new class of DG methods that use a local Virtual Element basis defined within each polygonal control volume. The basis functions are evaluated as an L^2 projection of the virtual basis, which remains unknown. Contrary to the Virtual Element Method (VEM) approach, the new basis functions lead to a nonconforming representation of the solution with discontinuous data across the element boundaries, as employed in DG discretizations.

The discretization in time is carried out following the ADER (Arbitrary order DERivative Riemann problem) methodology, which yields one-step fully discrete schemes that make use of a coupled space-time representation of the numerical solution.

The novel schemes are carefully validated against a set of typical benchmark problems for the compressible Euler and Navier-Stokes equations. The numerical results have been checked with reference solutions available in literature and also systematically compared, in terms of computational efficiency and accuracy, with those obtained by the corresponding modal DG version of the scheme.

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Fast space-time isogeometric solvers for nonlinear transient heat transfer problems

Thomas Elguedj

INSA-Lyon, LaMCoS

This talk explores the application of Space-Time IsoGeometric Analysis for solving nonlinear transient heat transfer problems and reviews numerical techniques aimed at enhancing computational efficiency. To address the nonlinear behavior of materials commonly encountered in industrial processes, we implement robust nonlinear solvers, including Newton's and Picard's methods. To mitigate the high computational costs and memory demands of space-time methods, we employ optimization strategies such as Matrix-Free and Weighted-Quadrature approaches. Additionally, we incorporate a Fast Diagonalization-based preconditioner to improve the efficiency of the iterative solver. Numerical experiments demonstrate the advantages of space-time methods over traditional incremental approaches, particularly when optimized algorithms are utilized. Our analysis also underscores the benefits of inexact nonlinear solvers in preventing oversolving and accelerating convergence. This study contributes to the development of efficient and scalable algorithms for nonlinear heat transfer problems.

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Robust space-time DG methods for the incompressible Navier-Stokes equations

Sergio Gomez

University of Milano-Bicocca

In this talk, we present the main ideas for the analysis of a space-time DG method for the incompressible Navier-Stokes equations. This method combines an H(div)-conforming DG spatial discretization with a DG time-stepping scheme, and we show that it is both pressure robust and Reynolds semi-robust.

Proving stability and convergence for high-order approximations requires nonstandard techniques. We overcome this challenge using carefully chosen test functions, establishing well-posedness, unconditional stability, and quasi-optimal error estimates (even for high Reynolds numbers).

Finally, we present numerical experiments that validate the robustness and accuracy of the method.

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A space-time multiscale mortar mixed finite element method for parabolic equations

Michel Kern

INRIA

We develop a space-time mortar mixed finite element method for parabolic problems. The domain is decomposed into a union of subdomains discretized with non-matching spatial grids and asynchronous time steps.

The method is based on a space-time variational formulation that couples mixed finite elements in space with discontinuous Galerkin in time. Continuity of flux (mass conservation) across space-time interfaces is imposed via a coarse-scale space-time mortar variable that approximates the primary variable. This setting allows for high flexibility with individual discretizations of each space-time subdomain, and in particular for local time stepping.

Uniqueness, existence, and stability, as well as a priori error estimates for the spatial and temporal errors are established. A key tool in the analysis is the construction of an interpolant in a space-time weakly continuous velocity space, which is used to prove a discrete divergence inf-sup condition on this space.

A space-time non-overlapping domain decomposition method is developed that reduces the global problem to a space-time coarse-scale mortar interface problem that can be solved with GMRES. Each interface iteration involves solving in parallel space-time subdomain problems. We prove a bound on the interface operator that leads to an estimate for the number of interface GMRES iterations through field-of-values analysis.

The method is implemented in a code based on the deal.ii parallel library. We illustrate the behavior of the method on several examples that highlight the advantages of the multiscale mortar space-time domain decomposition method.

Joint work with M. Jayadharan, M. Vohralik, and I. Yotov.

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Classical and optimized overlapping Schwarz methods within the finite volume framework

Stella Krell

University Cote d'Azur

I will present both classical and optimized overlapping Schwarz methods within the Finite Volume framework for the diffusion equation. The focus is on domain decomposition techniques specifically adapted to discretizations arising from finite volume schemes. I will begin by reviewing the classical Schwarz algorithm and providing several proofs of its convergence. In particular, I will present a convergence proof for the Two-Point Flux Approximation (TPFA) scheme based on the maximum principle, and another for the Discrete Duality Finite Volume (DDFV) method, relying on projection spaces and tools from functional analysis. In the second part of the talk, I will compare the performance of the classical Schwarz method with that of its optimized counterpart, which incorporates Ventcell transmission conditions. Numerical experiments will highlight the improved efficiency and robustness of the optimized approach across various test cases.

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Space-time finite element methods for the wave equation

Richard Löscher

TU Graz

In this talk, we discuss space-time formulations for the scalar wave equation. Although the time variable is treated as an additional dimension, a straightforward discretization of the variational formulation using conforming finite element spaces results in only conditional stability, which is already known from time-stepping methods as the CFL–condition.

To remedy this behavior, we consider the equivalent residual minimization problem, which leads to an unconditionally stable scheme. Furthermore, the minimization problem reveals an optimal transformation operator that, when applied to the test space, stabilizes the variational formulation.

We discuss numerical realizations and approximations of the transformation operator, as well as the stability of the resulting schemes when these transformations are applied. The theoretical findings are complemented by numerical examples.

Joint work with C. Köthe, O. Steinbach, and M. Zank.

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Parareal and IETI-DP for parabolic PDEs with vanishing time derivatives on certain subdomains

Mario Mally

CEM at TU Darmstadt and mat+i at University of Santiago de Compostela

Our work focuses on parabolic PDEs with a partially vanishing time derivative. Electromagnetic eddy current problems are a prominent example because some regions are non-conducting, which implies that the time derivative vanishes there. In combination with the curl operator, simulations become cumbersome due to its nontrivial kernel.

In this context, we explore the use of IETI-DP to improve scalability and decrease computational time. An important tool is the tree-cotree gauge, which is used to enable concurrent computations for non-conducting subdomains by eliminating the kernel of the curl operator on a discrete level. Additionally, we investigate an extension with the nonintrusive Parareal algorithm to also exploit concurrent computations in time.

Application-oriented numerical simulations are carried out to verify analytically derived properties of the proposed algorithms and as a proof of concept.

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An hp multigrid approach for tensor-product space-time finite element discretizations

Nils Margenberg

Helmut Schmidt University

We develop a space-time multigrid method within the framework of space-time finite element methods. The approach includes both continuous and discontinuous variational time discretizations of high order, combined with continuous spatial finite element discretizations. Multigrid methods have been proven to be efficient for large-scale problems. However, extending these advantages to the space-time domain presents challenges. A critical part to achieve good performance is an effective smoother. We employ a space-time cell-wise Additive Schwarz smoother and consider further developments to reduce its computational cost. We demonstrate its effectiveness for the acoustic wave equations, convection-diffusion-equations and the Stokes equations.

We discuss the efficient implementation of space-time multigrid methods using the matrix-free framework provided by the dealii finite element library. Our implementation supports h, p, and hp-Multigrid strategies across both space and time dimensions. We perform scaling and convergence tests on high-performance computing platforms. The method is tested on unstructured meshes and problems with heterogeneous coefficients.

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Space-time approximation of the wave equation with splines in time

Ilaria Perugia

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We present a conforming space–time method for the wave equation based on a novel second-order-in-time variational formulation. Inspired by the continuous/discontinuous Galerkin time-stepping approach introduced by [1], the method is derived by discretizing an associated ODE in time, with stability ensured through two main components: modifying the test functions via an appropriate isomorphism, and including a term that involves the first time derivative at the initial time. This results in a coercive variational formulation with exponential-weighted inner products. Both trial and test functions are discretized using splines with at least C1 regularity. The convergence is proven to be suboptimal by one order in general, but optimal for C1-splines of even degree. Numerical results suggest that quasi-optimal convergence occurs when the difference between degree and regularity is odd, including the case of splines with maximal regularity.

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Isogeometric analysis in space-time

Giancarlo Sangalli

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Isogeometric Analysis (IGA) uses splines and extensions (like NURBS) as basis functions, aligning numerical simulation closely with computer-aided design and offering higher accuracy than traditional FEM. A natural extension is to use splines in the time domain for evolutionary PDEs. Spline-based temporal discretization opens possibilities for parallel-in-time solvers: we present efficient solvers that exploit the tensor-product structure of spline spaces using tensor linear algebra. This talk will explore space-time IGA for both parabolic and hyperbolic PDEs, highlighting benefits, limitations, and computational considerations.

Joint work with S. Fraschini, G. Loli, A. Moiola, M. Montardini, and M. Tani.

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Space-time single-goal and multi-goal a posteriori error control for adaptively balancing discretization and numerical solver errors

Thomas Wick

Leibniz University Hannover

This talk is on goal-oriented a posteriori error control, adaptivity and solver control for finite element approximations to boundary and initial-boundary value problems for stationary and non-stationary partial differential equations, respectively. In particular, coupled field problems with different physics may require simultaneously the accurate evaluation of several quantities of interest, which is achieved with multi-goal oriented error control. Sensitivity measures are obtained by solving an adjoint problem. Error localization is achieved with the help of a partition-of-unity. The resulting adaptive algorithms allow to balance discretization and non-linear iteration errors, and are demonstrated for some numerical examples.

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Conforming space-time methods for time-dependent Schrödinger equations

Marco Zank

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The approximation of the solutions to time-dependent partial differential equations is usually based on time-stepping schemes and spatial discretizations, such as finite element or finite difference methods. Space-time methods take another path, interpreting the temporal variable as another spatial variable. For this purpose, a space-time mesh as a decomposition of the space-time cylinder is needed, and the trial and test functions are functions in space and time. In addition, space-time methods lead to a linear system, which has to be solved at once.

In this talk, we consider the linear time-dependent Schrödinger equation with Dirichlet boundary conditions. First, we examine the space-time variational setting, including its solvability. Second, we derive a conforming space-time finite element method, which is based on piecewise polynomial, globally continuous trial and test functions. In particular, we investigate a tensor-product approach concerning the temporal and spatial variables. Finally, we give numerical examples for a two-dimensional spatial domain.

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MS09 – Fast solution techniques for polytopal methods and related applications: a Nemesis mini-symposium

Organizers: Paola Antonietti, Lourenco Beirao da Veiga, Daniele A. Di Pietro, Jerome Droniou

Finite element methods are among the most widely used discretisation techniques for solving models governed by partial differential equations (PDEs). Over the past decade, polytopal methods—finite element discretisations that accommodate significantly more general meshes than standard finite elements—have emerged as powerful tools for addressing the complexities of differential systems, thanks to their remarkable flexibility in defining the discrete spaces. Various successful polytopal methods have been developed, including e.g., (listed alphabetically) the Discrete de Rham (DDR), Hybrid High-Order (HHO), polytopal Discontinuous Galerkin (polyDG), and the Virtual Element Method (VEM). This mini symposium aims at bringing together researchers who are actively developing polytopal and related methods to address real-world problems. The focus will encompass theoretical and practical perspectives, including the development of efficient computational strategies. Special attention will be given to recent advancements in constructing fast solution algorithms and scalable preconditioners, which are crucial for solving the algebraic systems that arise from polytopal discretisations of multiphysics and multiscale PDEs.

The organisers of this mini symposium acknowledge the funding of the European Union (ERC Synergy, NEMESIS, project number 101115663). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

List of Speakers

- S. Berrone (Politecnico di Torino) – *Stabilisation-free VEM: definitions, construction details, and a priori/a posteriori error estimates for low- and high-order methods.*
- S. Bonetti (Politecnico di Milano) – *Unified primal and mixed discontinuous Galerkin analysis of non-isothermal Darcy-Forchheimer flows.*
- A. Cancrini (Politecnico di Milano) – *A polytopal discontinuous Galerkin method for the pseudo-stress formulation of the unsteady Stokes problem.*
- F. Credali (King Abdullah University of Science and Technology) – *The reduced basis multigrid scheme for the virtual element method.*
- F. Dassi (University Milano - Bicocca) – *Parallel solvers for virtual element discretizations of elliptic equations in mixed form.*
- L. Helthai (Università di Pisa) – *Effortless Hierarchies for Complex 3D Meshes: unlocking Geometric Multigrid with R3MG and discontinuous Galerkin polytopic methods.*
- M.-Y. Kim (Inha University) – *An edgewise iterative scheme for the discontinuous Galerkin method with Lagrange multiplier for Poisson's equation.*

- S. Scacchi (Università degli Studi di Milano) – *Conditioning and nonoverlapping domain decomposition algorithms for* C^1 *virtual element discretizations of the biharmonic equation*.

Stabilisation-free VEM: definitions, construction details, and a priori/a posteriori error estimates for low- and high-order methods

Stefano Berrone

Politecnico di Torino

This presentation introduces and analyses a stabilisation-free Virtual Element Method (VEM) for the numerical solution of second-order elliptic equations. The core innovation of the method lies in the use of novel polynomial projections that allow for the design of structure-preserving schemes, thereby eliminating the need for traditional stabilization terms.

Stabilisation has long been a central topic in the development of VEM, with extensive research devoted to its analysis and implementation. Recently, stabilisation-free approaches have gained growing attention, particularly in applications involving non-isotropic diffusion, nonlinear elasticity, elastoplasticity, and a posteriori error estimation.

We will provide a detailed discussion of the construction and implementation of the stabilisation-free VEM scheme, highlighting its robustness, especially in handling anisotropic problems, and its capacity to yield a posteriori error estimates that are free from the often problematic influence of stabilisation parameters in both lower and upper bounds.

Several numerical experiments will be presented to illustrate the stability and accuracy of the method in solving challenging problems characterised by anisotropic diffusion.

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Unified primal and mixed discontinuous Galerkin analysis of non-isothermal Darcy-Forchheimer flows

Stefano Bonetti

Politecnico di Milano

In this talk, we present and analyze two schemes for the numerical modeling of a Darcy-Forchheimer fluid flow model coupled with an advection diffusion equation for describing the temperature distribution inside a fluid. We propose a discontinuous Galerkin(dG)-dG-dG scheme that is appealing for handling polytopal grids and arbitrary-order approximations and a Raviart-Thomas(RT)-dG-dG scheme that allows to carry out numerical simulations without dramatically affecting the computational cost, feature that is of particular interest for three-dimensional simulations. A fixed-point linearization strategy – which naturally induces a splitting solution strategy – is adopted for treating the non-linearities of the problem. For the discrete problem, we develop the stability analysis and we prove the convergence of the iterative algorithm under mild requirements on the problem data and independent with respect to the adopted scheme. Finally, a set of numerical simulations is presented in order to assess the convergence properties of the proposed schemes and to test their performance in physically sound test cases, both in two-dimensional and three-dimensional settings.

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A polytopal discontinuous Galerkin method for the pseudo-stress formulation of the unsteady Stokes problem

Alessandra Cancrini

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In this talk, we present the development and analysis of a discontinuous Galerkin method on polytopal grids (PolydG) for the unsteady Stokes problem written in its pseudo-stress formulation. Due to the recent growing interest in non-Newtonian fluid flow models, which are crucial for modeling significant processes in the biological, medical, and industrial fields, the stress-velocity-pressure formulation for incompressible flows has garnered attention. Indeed, for complex non-linear flow problems, the use of a formulation where stress serves as a primal unknown can facilitate the design of approximation methods and its numerical solution. In addition, an accurate approximation of the stress is crucial for determining traction on a fluid-solid interface. Although stress can be reconstructed a posteriori in the velocity-pressure formulation through velocity differentiation, this compromises the accuracy. Note that a drawback associated with employing the stress-velocity-pressure formulation is the additional challenge introduced by the symmetry constraint of the stress tensor during the discretization process. One possible approach to overcome such a difficulty is based on employing the concept of pseudo-stress. The pseudo-stress, being nonsymmetric, allows for the adaptation of stable pairs designed for Darcy flows to the pseudostress-velocity formulation of the Stokes equations. For this reason, we formulate the unsteady Stokes problem as a single equation in the pseudo-stress variable. Our work introduces a new contribution by presenting a comprehensive analysis of the proposed PolydG approximation of the Stokes problem in its pseudo-stress formulation. First, we provide rigorous proof of the well-posedness and stability of the pseudo-stress weak formulation of the continuous problem, which is both novel and original. We then design and analyze both the semi-discrete and fully-discrete formulations based on the PolydG spatial discretization and the theta-method time integration, carrying out a detailed stability analysis for both, and obtaining a priori estimates. Finally, we present a convergence analysis for the fully-discrete problem, establishing error estimates in a suitable discrete norm. Additionally, we present a set of verification tests to validate the theoretical results and apply the method to cases of engineering interest in two- and three-dimensional settings.

Joint work with P.F. Antonietti, M. Botti, and I. Mazzieri.

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The reduced basis multigrid scheme for the virtual element method

Fabio Credali

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Virtual element methods (VEM) are a family of polytopal numerical methods for the approximation of PDEs. The adjective "virtual" refers to the fact that the basis functions of the underlying discrete space are not explicitly known since they are solutions of local PDEs. Thus, their evaluation is not generally required and several quantities are computed by means of polynomial projections.

The virtual nature of the VEM space is an issue when designing multigrid solvers since the definition of intergrid operators requires the evaluation of the basis functions at points in the elements interior. A first solution to this issue was the evaluation of polynomial projections [1].

We propose a Reduced Basis approach to efficiently solve the local equation associated to each virtual basis function [2]. Thus, we define a new discrete space (rbVEM) with known basis functions and having the same properties of the original VEM space. We exploit the rbVEM space to design efficient geometric multigrid schemes.

Joint work with P.F. Antonietti and S. Bertoluzza.

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Parallel solvers for virtual element discretizations of elliptic equations in mixed form

Franco Dassi

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In the last decades, a large variety of methods for the approximation of partial differential equations on polytopal meshes have been developed. Among them Discontinuos Galerkin (DG), Hybrid High-Order methods (HHO) and the Virtual Element Method (VEM) are the most common ones. Such novel methodologies pose new challenges in the numerical resolution of linear systems arising from the discretization of the partial differential equation at hand.

In this talk, we focus on the resolution of the mixed formulation of three-dimensional elliptic equations via VEM. After a first description of the virtual element method itself, we move to the linear algebra. Indeed, we make a deep numerical investigation on the behaviour of both direct and iterative parallel solvers for these type of saddle-point linear systems. Moreover, we analyze the performace of different types of block preconditioners.

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Effortless Hierarchies for Complex 3D Meshes: unlocking Geometric Multigrid with R3MG and discontinuous galerkin polytopic methods

Luca Helthai

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R3MG (R-tree-Based Multigrid) is an automated, robust, and dimension-independent approach that generates nested and balanced hierarchies of agglomerates, starting from arbitrary polygonal and polyhedral grids. By leveraging the spatial indexing properties of R-trees, this technique allows the robust and automated generation of nested hierarchies of polytopal agglomerates without user intervention, that preserve mesh quality and improve computational efficiency, outperforming traditional graph-based methods like METIS in structured, unstructured, and polytopal grid scenarios. Applications include the development of robust grid hierarchy generation, in view of optimal geometric multigrid preconditioners for discontinuous Galerkin methods.

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An edgewise iterative scheme for the discontinuous Galerkin method with Lagrange multiplier for Poisson's equation

Mi-Young Kim

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An edgewise iterative scheme is developed for large systems of equations resulting from the discretization by the discontinuous Galerkin method with Lagrange multiplier for the Poisson's equation. The solution is computed element by element. Lagrange multiplier is edgewise updated, which is given as the average of the Robin type information on the elements sharing the edge. Analysis of the convergence of the scheme is given with the discrete maximum norm over all the edges. Several numerical experiments are presented.

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Conditioning and nonoverlapping domain decomposition algorithms for C^1 virtual element discretizations of the biharmonic equation

Simone Scacchi

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The virtual element method (VEM) has been proven to provide effective arbitrarily regular Galerkin approximations of elliptic partial differential equations of order $2r$, for any integer r larger than or equal to 1. Here we focus on the two-dimensional biharmonic equation, that corresponds to the case $r = 2$. The proposed VEM approximation space is globally C^1, with the advantage of being conforming in H^2 and making use of a very simple set of degrees of freedom (dofs), namely, 3 dofs per vertex of the mesh. We first investigate how different VEM stabilization strategies influence the conditioning of the stiffness matrix and the convergence of the scheme. Then, we develop a balancing domain decomposition by constraints (BDDC) preconditioner for the proposed discretization and we study numerically the optimality and scalability of the resulting iterative algorithm.

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MS10 – Scalable domain decomposition solvers for cellular reaction diffusion models in computational biology

Organizers: Edoardo Centofanti, Ngoc Mai Monica Huynh, Simone Scacchi

Cell-by-cell models, which describe the behavior of individual cells and their interactions, can be found in several applications, e.g. in cardiac electrophysiology and neuroscience. These models are characterized by high-dimensional and often nonlinear systems of equations. Besides the need of accuracy and biological fidelity from a modeling perspective, the computational workload of solving such models remains a significant challenge, particularly for simulations involving millions of cells. This session focuses on recent advances in scalable solvers designed to address these challenges. Key themes include parallelization strategies, domain decomposition methods, and machine learning-accelerated solvers. Particular attention is given to approaches that balance computational efficiency with the need for high accuracy in capturing cell-level dynamics and tissue-level emergent behavior. The session will feature talks on (but not limited to) innovative algorithms leveraging High-Performance Computing infrastructures, enabling researchers to fastly solve cell-by-cell models with high orders of magnitude. By bringing together experts in computational biology, applied mathematics, and high-performance computing, this session aims to foster collaboration and inspire new directions for scalable solutions, encouraging discussions to deeper understand the available state-of-the-art tools, practical implementation strategies, and the opportunities they unlock for advancing large-scale biological simulations.

List of Speakers

- T. Abdelhamid (University of Milan) – *A Balancing domain decomposition by constraints for high-performance bidomain solvers on CPUs*.
- P. Benedusi (Università della Svizzera Italiana) – *Scalable approximation and solvers for ionic electrodiffusion in cellular geometries*.
- S. Botti (Università della Svizzera Italiana) – *Steklov-Poincaré domain decomposition for hiPSC-CMs multi-electrode arrays models*.
- Z. Lin (Shenzhen Institutes of Advanced Technology) – *A highly parallel finite element method for modelling the Darcy flow in human liver tissue*.
- L.F. Pavarino (Università degli Studi di Pavia) – *Generalized Dryja–Smith–Widlund (GDSW) preconditioners for composite discontinuous Galerkin discretizations of multicompartment reaction–diffusion problems*.
- V. Pederzoli (Politecnico di Milano) – *A coupled mathematical and numerical model for protein spreading and tissue atrophy applied to Alzheimer's disease*.
- M. Sarkis (Worcester Polytechnic Institute) – *NOSAS for the cardiac cell-by-cell model*.
- S. Serra-Capizzano (University of Insubria and University of Uppsala) – *Updating the GLT analysis: new tools, applications, and beyond*.

A balancing domain decomposition by constraints for high-performance bidomain solvers on CPUs

Talaat Abdelhamid

University of Milan

The numerical simulation of cardiac electrophysiology presents a considerable challenge in scientific computing. The bidomain model, which is the most comprehensive mathematical representation of cardiac bioelectricity, consists of an elliptic and a parabolic partial differential equation (PDE) of reaction-diffusion type. These equations govern the transmission of electrical signals within cardiac tissue and are strongly coupled with a rigid system of ordinary differential equations (ODEs) that describe ionic currents across the cardiac cell membrane. Minimizing the computational costs of these simulations requires the development of efficient and scalable preconditioners for the linear systems resulting from the model's discretization. This study examines the Bidomain system as a representative problem and evaluates the performance of three preconditioner implementations PCBDDC, PCBJACOBI, and Geometric Algebraic Multigrid (GAMG) available in the PETSc library. The study specifically analyzes their effectiveness in relation to various parameter tuning strategies. Our analysis was performed on modern high-performance computing (HPC) architectures, where we conducted scalability tests in multicore environments. Different refined meshes are generated with the fiber data for each mesh. Each mesh was partitioned into different subdomains (i.e., 4, 8, 16, 32, 64), each subdomain mapped to a single processor, ensuring a one-to-one correspondence between processes and subdomains. We used METIS to partition the mesh elements and developed in-house code for managing the local-to-global mapping. The results highlight the efficiency and accuracy of the implemented preconditioners, confirming their scalability on CPUs. This paper presents an optimized solution strategy that leverages the preconditioned Balancing Domain Decomposition by Constraints (PCBDDC) method. The performance of the PCBDDC preconditioner is evaluated on CPU architectures and compared against other commonly used preconditioners AMG Hyper and GAMG. The results demonstrate significant improvements in computational efficiency and scalability, positioning them as a promising option for large-scale cardiac simulations.

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Scalable approximation and solvers for ionic electrodiffusion in cellular geometries

Pietro Benedusi

Università della Svizzera Italiana (USI)

The activity and dynamics of excitable cells are fundamentally regulated and moderated by extracellular and intracellular ion concentrations and their electric potentials. The increasing availability of dense reconstructions of excitable tissue at extreme geometric detail pose a new and clear scientific computing challenge for computational modeling of ion dynamics and transport. We introduce a scalable numerical algorithm for solving the time-dependent and nonlinear KNP-EMI (Kirchhoff-Nernst-Planck Extracellular-Membrane-Intracellular) equations describing ionic electrodiffusion for excitable cells with an explicit geometric representation of intracellular and extracellular compartments and interior interfaces. Our solution strategy is based on a mixed finite element discretization of ion concentrations and electric potentials in intracellular and extracellular domains and an algebraic multigrid-based, inexact block-diagonal preconditioner for GMRES. Such a solution strategy is motivated and studied via spectral analysis of the corresponding discrete operators. Numerical experiments with up to 100M unknowns per time step and up to 256 cores demonstrate that this solution strategy is robust and scalable with respect to the problem size, time discretization and number of cores.

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Steklov-Poincaré domain decomposition for hiPSC-CMs multi-electrode arrays models

Sofia Botti

Università della Svizzera Italiana

Multi-electrode arrays (MEAs) enable the electrophysiological study of several cell cultures, among which human induced pluripotent stem cell-derived cardiomyocytes (hiPSC-CMs), by capturing extracellular potentials, offering a non-invasive platform to study tissue-level cardiac behavior. Bridging these experimental observations with computational models is crucial for advancing in-silico cardiac research. In this work, we present a computational framework that couples MEA-based modeling with the bidomain equations to simulate the electrical activity of heterogeneous hiPSC-CM tissues. Due to the high resolution required to reproduce realistic extracellular signals and spatial heterogeneity, these simulations pose significant computational challenges. To address this, we employ a non-overlapping domain decomposition strategy based on the Steklov-Poincaré formulation, providing a modular and scalable solver framework well-suited for parallel high-performance computing environments. We analyze the convergence behavior of the Conjugate Gradient (CG) method used to solve the interface problem, focusing on the number of iterations required to reach a fixed tolerance. This is studied as a function of the mesh size h and the subdomain size H, both without and with preconditioning. Furthermore, we propose a tailored preconditioner designed to enhance convergence rates and reduce computational effort in large-scale simulations. While this work represents a preliminary study, it establishes a foundation for scalable simulation of MEA-integrated bidomain models in hiPSC-CM tissues, with potential applications in virtual electrophysiological testing and computational cardiology.

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A highly parallel finite element method for modelling the Darcy flow in human liver tissue

Zeng Lin

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Liver diseases primarily occur in liver tissue, so the numerical simulation of blood flow in liver tissue holds paramount significance. Nevertheless, there is a scarcity of research dedicated to the numerical simulation of porous media flow in liver tissue, particularly in the context of realistic 3D human liver. In this work, we introduce a highly parallel finite element method designed for the transient Darcy equation, specifically tailored for simulating the blood flows in 3D patient-specific liver tissue. The parallel and stabilized finite element methods on fully unstructured meshes are described in detail for solving the 3D transient Darcy equation. For decreasing the number of iterations and improving the parallel efficiency, the Krylov-Schwarz (KS) algorithm is innovatively utilized to resolve the discretized system. Several numerical examples for mono-permeability and multi-permeability are studied to validate the accuracy and scalability of the proposed methodology. Moreover, a parallel efficiency near 60% is attained in the resolution of the transient Darcy problem involving 38.93 million elements, utilizing a supercomputer equipped with 4800 processor cores.

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Generalized Dryja–Smith–Widlund (GDSW) preconditioners for composite discontinuous Galerkin discretizations of multicompartment reaction–diffusion problems

Luca Franco Pavarino

Università degli Studi di Pavia

This talk focuses on the design, analysis, and numerical study of a generalized Dryja–Smith–Widlund (GDSW) preconditioner for composite Discontinuous Galerkin discretizations of multicompartment parabolic reaction–diffusion equations, where the solution can exhibit natural discontinuities across the domain [1]. We prove that the resulting preconditioned operator for the solution of the discrete system arising at each time step converges with a scalable and quasi-optimal upper bound for the condition number. The GDSW preconditioner is then applied to the EMI (Extracellular - Membrane - Intracellular) reaction–diffusion system, recently proposed to model microscopically the spatiotemporal evolution of cardiac bioelectrical potentials. Numerical tests validate the scalability and quasi-optimality of the EMI-GDSW preconditioner, and investigate its robustness with respect to the time-step size as well as jumps in the diffusion coefficients.

[1] N.M.M. Huynh, L.F. Pavarino, and S. Scacchi. GDSW preconditioners for composite Discontinuous Galerkin discretizations of multicompartment reaction–diffusion problems. *Comput. Methods Appl. Mech. Eng.* (2025).

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A coupled mathematical and numerical model for protein spreading and tissue atrophy applied to Alzheimer's disease

Valentina Pederzoli

Politecnico di Milano

In this talk, we present a new mathematical model describing the interplay between biological tissue atrophy driven by the diffusion of a biological agent and its applications to neurodegenerative diseases. The model of study comprises a Fisher-Kolmogorov equation for species diffusion, incorporating both dispersion and proliferation effects, coupled with an elasticity equation governing tissue atrophy. This mass loss process is described through a morpho-elastic framework, where mass loss and tissue elasticity together shape the resulting tissue morphology. The model integrates the morpho-elastic response with pathogen concentration by introducing an evolution law for inelastic strain, governed by pathogen concentration through a logistic-type differential equation. We present the application of this model to the onset and development of Alzheimer's disease, where the equations describe the propagation of misfolded τ-proteins and the ensuing brain atrophy characteristic of the disease. To address the inherited complexities numerically, we propose a Discontinuous Galerkin (DG) method for spatial discretization, while time integration relies on the Crank-Nicolson method. We present convergence tests to validate the DG method in the uncoupled case, discuss the results concerning the theoretical outcomes and validate the model in the coupled framework. Moreover, we have applied the model to simulate Alzheimer's disease on a real brain geometry, where we observed outcomes consistent with the anticipated biological behaviour of prion-like protein diffusion and tissue atrophy. As a proof of concept, we also present the results of a simulation incorporating a nonlinear constitutive law for tissue elasticity.

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NOSAS for the cardiac cell-by-cell model

Marcus Sarkis

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We introduce the Non-Overlapping Spectral Additive Schwarz (NOSAS) methods for the cardiac cell-by-cell model. The discretizations are very similar to the composite Discontinuous Galerkin (DG) discretization. We consider two spectral versions of the NOSAS, one based on zero extension inside the subdomains and the other one based on the block-diagonal of the local Schur complement. Theorems and numerical results will be presented.

Joint work with N.M.M. Huynh, L.F. Pavarino, and S. Scacchi.

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Updating the GLT analysis: new tools, applications, and beyond

Stefano Serra-Capizzano

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Various classes of Generalized Locally Toeplitz (GLT) sequences have been introduced as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators and, for every sequence of one the given classes, it has been demonstrated that it is possible to give a rigorous description of the asymptotic spectrum in terms of a function (the symbol) that can be easily identified. The latter generalizes the notion of a symbol for differential operators (discrete and continuous) or for Toeplitz sequences, where for the latter it is identified through the Fourier coefficients and is related to the classical Fourier Analysis.

For every $r, d \geq 1$ the r-block d-level GLT class has nice $*$-algebra features and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol $=$ a symbol whose minimal singular value vanishes at most in a set of zero Lebesgue measure). Furthermore, the GLT $*$-algebras virtually include any approximation of partial differential equations (PDEs), fractional differential equations (FDEs), integro-differential equations (IDEs) by local methods (Finite Difference, Finite Element, Isogeometric Analysis etc) and, based on this, we demonstrate that our results on GLT sequences can be used in a PDE/FDE/IDE setting in various directions, including multi-iterative solvers (combining preconditioned Krylov, multigrid, etc), spectral detection of branches, fast 'matrix-less' computation of eigenvalues, stability issues. We will discuss also the impact and the further potential of the theory with special attention to new tools and to new directions as those based on symmetrization tricks, on the extra-dimensional approach, and on block structures or blocking operations, as it occurs in Domain Decomposition techniques or dealing with systems of PDEs.

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MS11 – Robust parallel solvers for linear, nonlinear, and multiphysics problems

Organizers: Shihua Gong, Wei Wang, Chensong Zhang

The development of robust and scalable parallel solvers is a critical component of modern scientific computing, enabling the efficient simulation of complex systems across a wide range of disciplines, including physics, engineering, climate modeling, and biomedical applications. This minisymposium will focus on advances in algorithms and software for solving linear, nonlinear, and multiphysics problems on high-performance computing platforms.

Topics of interest include, but are not limited to, preconditioners for large-scale linear systems, iterative solvers for nonlinear equations, domain decomposition methods, and frameworks for coupling multiple physical models. Special emphasis will be placed on approaches that achieve robust convergence, ensure good scalability, and adapt to modern computing architectures.

Speakers will present innovative strategies for addressing challenges such as ill-conditioned systems, strong coupling in multiphysics problems, and large-scale eigenvalue computations in parallel environments. Applications in areas such as wave propagation, porous media flow, and coupled electromagnetic simulations will illustrate the practical impact of these advances.

This minisymposium aims to bring together researchers and practitioners to exchange ideas, share experiences, and foster collaboration on the design of high-performance solvers that address the growing demands of computational science and engineering.

List of Speakers

- H. Chen (Xiamen University) – *Thermodynamically consistent modeling and simulations for compressible flow in porous media*.
- S. Huang (Institute of Applied Physics and Computational Mathematics) – *An adaptive setup-based two-level preconditioner for linear systems arising from radiation diffusion equations*.
- B. Jiang (KAUST) – *Connections between subspace correction methods and convex optimization algorithms*.
- Q. Liang (Tongji University) – *Overlapping Schwarz Methods in* $H(\mathrm{curl};\Omega)$.
- J. Park (King Abdullah University of Science and Technology) – *Subspace correction methods for nearly semicoercive problems and an application to the Forchheimer model*.
- P. Piersanti (The Chinese University of Hong Kong) – *Numerical approximation of variational inequalities in elasticity: theory, libraries and counterexamples*.
- Z. Tan (Xiamen University) – *A finite element method for a two-dimensional Pucci equation*.
- S. Wu (Peking University) – *A robust solver for* $H(\mathrm{curl})$ *convection-diffusion and its local Fourier analysis*.
- Z. Yan (Shenzhen Institutes of Advanced Technology) – *Efficient domain decomposition methods for patient-specific cardio-cerebral hemodynamics simulations*.

- Y. Yang (Xiangtan University) – *Multiscale modeling of wave propagation with exponential integration and GMsFEM*.
- C. Zhang (Academy of Mathematics and Systems Science) – *Learning-based multilevel solvers for large-scale linear systems*.
- B. Zheng (Chinese Academy of Science) – *Two-level hybrid domain decomposition method for Helmholtz problem with high wave number*.

Thermodynamically consistent modeling and simulations for compressible flow in porous media

Huangxin Chen

Xiamen University

Modeling and simulation of compressible flow in porous media are of great interest in the fields of hydrology and petroleum reservoir engineering. In this talk, we will introduce a thermodynamically consistent mathematical model for compressible flow in porous media with rock compressibility and in poroelasticity media, and the energy-stable and conservative algorithms will be discussed. For the non-isothermal compressible flow in porous media, we will also discuss an energy conservative and entropy stable numerical scheme.

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An adaptive setup-based two-level preconditioner for linear systems arising from radiation diffusion equations

Silu Huang

Institute of Applied Physics and Computational Mathematics

In the numerical simulation process of laser fusion, as well as other complex applications, it is essential to solve sparse linear systems of radiation diffusion equations. In general, radiation diffusion equations can be discretized in space using the finite volume approach and linearized using the Picard or Newton method, leading to a sequence of sparse linear systems whose count surely reaches several tens of thousands, hundreds of thousands, or even millions. When dealing with a sequence of linear systems whose properties change dynamically as the application features evolve dynamically, the general solvers completely ignore the differences among linear systems in the sequence and use a fixed strategy for all linear systems, making the solution inefficient as well. In this talk, an adaptive setup-based preconditioner is presented, which selects the appropriate preconditioner for each linear system in sequence to increase overall performance.

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Connections between subspace correction methods and convex optimization algorithms

Boou Jiang

KAUST

The framework of subspace correction methods connects a broad range of iterative algorithms, from elementary Jacobi and Gauss–Seidel methods to advanced multigrid and domain decomposition methods, offering a systematic approach for their design and analysis. In this paper, we extend these connections to various classes of convex optimization algorithms, including alternating projection methods, operator splitting methods, and multiplier methods. More precisely, we show that these algorithms can be derived from subspace correction methods via convex duality, which relates two optimization problems, known as the primal and dual problems. To this end, we introduce the concept of *dualization*, a process that transforms an iterative method for solving the dual problem into an equivalent method for solving the primal problem. While dualization provides a unified understanding of well-known equivalences among existing convex optimization algorithms, it also establishes new connections among subspace correction, alternating projection, operator splitting, and multiplier methods. In particular, we show that the von Neumann, Dykstra, Peaceman–Rachford, and Douglas–Rachford algorithms can be interpreted as dualizations of subspace correction methods. We also establish a dualization relation between alternating direction methods of multipliers and operator splitting algorithms. This perspective provides a unified viewpoint for these algorithms, enabling the transfer of state-of-the-art results and promoting the development of new algorithms and theoretical insights.

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Overlapping Schwarz Methods in $H(\mathrm{curl};\Omega)$

Qigang Liang

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The overlapping Schwarz method is one of the most important methods for solving large-scale discrete problems arising from partial differential equations. The previous proved bound is $C(1+H^2/\delta^2)$ for the condition number of the overlapping domain decomposition methods in $H(\mathrm{curl};\Omega)$, where H and δ are the sizes of subdomains and overlaps, respectively. However, all numerical results indicate that the best bound is $C(1+H/\delta)$. In this talk, we shall solve this long-standing open problem by proving that is indeed the best bound. Based on the overlapping Schwarz methods, we shall propose a two–level preconditioned Helmholtz subspace iterative (PHSI) method for solving algebraic eigenvalue problems resulting from edge element approximation of Maxwell eigenvalue problems. The two-level PHSI method may compute simple eigenpairs, multiple eigenpairs and clustered eigenpairs.

Joint work with X. Xu and S. Zhang.

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Subspace correction methods for nearly semicoercive problems and an application to the Forchheimer model

Jongho Park

King Abdullah University of Science and Technology

We present a convergence theory of subspace correction methods for nearly semicoercive convex optimization problems, extending the theory of nearly singular linear problems to the nonlinear setting. More precisely, we establish a parameter-independent convergence rate under the assumption that the kernel of the semicoercive part can be decomposed into a sum of local kernels, consistent with the linear case. As an application, we propose parallel multilevel methods for the Forchheimer model, which describes the flow of a high-velocity fluid through a porous medium. We first reformulate the Forchheimer model as a nearly semicoercive convex optimization problem using the augmented Lagrangian method. Then, by applying the framework of subspace correction methods for nearly semicoercive convex optimization problems, we develop robust parallel multilevel methods.

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Numerical approximation of variational inequalities in elasticity: theory, libraries and counterexamples

Paolo Piersanti

The Chinese University of Hong Kong, Shenzhen

In solid mechanics, shells are three-dimensional structures of small thickness compared to the extension they cover. Such structures are abundant in nature (eggs, snails, turtles, blood vessels,...) but also in industry (ship hulls, plane fuselage, roofs, glasses, tires...). One of the reasons for this popularity is because of their ability to sustain applied loads in a very effective way, with the minimum amount of material they require, and the lightness and economy that this represents.

One of the most popular models for studying the deflection of a linearly elastic shell is Koiter's model. Koiter's model is a two-dimensional model (two-dimensional, in the sense that it is defined over a two-dimensional subset of the Euclidean plane) that was formulated by solely resorting to assumption of geometrical (Kirchhoff—Love) and mechanical (Fritz John) nature. Koiter's model was proved to be, in the obstacle-free case [1], an adequate replacement for the standard equations of three-dimensional linearized elasticity. The latter discovery allowed practitioners to avoid dealing with the locking phenomenon when implementing numerical simulations for shells models.

In this talk, we review some recent contributions concerning the convergence of Finite Element Methods for Koiter's model for linearly elastic shells constrained to remain confined in a prescribed half space. We analyse shells characterized by different modes of deformations, and we present numerical results based on a library developed by the speaker in collaboration with several Authors. We will also examine the special cases of shallow shells, for which the solution cannot be discretized - even by resorting to a Mixed Formulation - without opting for finite elements of class C^1.

References

[1] P.G. Ciarlet, V. Lods, and B. Miara. Asymptotic analysis of linearly elastic shells. II. Justification of flexural shell equations. *Arch. Rational Mech. Anal.* (1996).

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A finite element method for a two-dimensional Pucci equation

Zhiyu Tan

Xiamen University

A nonlinear least-squares finite element method for strong solutions of the Dirichlet boundary value problem of a two-dimensional Pucci equation on convex polygonal domains is investigated in this talk. We obtain a priori and a posteriori error estimates and present corroborating numerical results, where the discrete nonsmooth and nonlinear optimization problems are solved by an active set method and an alternating direction method with multipliers.

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A robust solver for $H(\mathrm{curl})$ convection-diffusion and its local Fourier analysis

Shuonan Wu

Peking University

We present a robust and efficient multigrid solver based on an exponential-fitting discretization for 2D $H(\mathrm{curl})$ convection-diffusion problems. By leveraging an exponential identity, we characterize the kernel of $H(\mathrm{curl})$ convection-diffusion problems and design a suitable hybrid smoother. This smoother incorporates a lexicographic Gauss-Seidel smoother within a downwind type and smoothing over an auxiliary problem, corresponding to H(grad) convection-diffusion problems for kernel correction. We analyze the convergence properties of the smoothers and the two-level method using local Fourier analysis (LFA). The performance of the algorithms demonstrates robustness in both the convection-dominated and diffusion-dominated cases.

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Efficient domain decomposition methods for patient-specific cardio-cerebral hemodynamics simulations

Zhengzheng Yan

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In clinical practice, some patients experience life-threatening strokes during coronary revascularization, a common intervention for coronary artery disease. Despite its clinical significance, few computational studies have investigated how revascularization affects cardio-cerebral hemodynamics. To address this, we reconstructed a patient-specific three-dimensional cardio-cerebral arterial network with coronary stenosis and evaluated the hemodynamic consequences of stenosis removal. The time-dependent incompressible Navier–Stokes equations were discretized using a stabilized P1–P1 Galerkin finite element method with an implicit second-order backward differentiation formula (BDF2). A regional blood flow distribution model, coupled with lumped Windkessel boundary conditions, was applied at the outlets. The large-scale 3D pulsatile flow problem was solved using a scalable parallel solver based on a Newton–Krylov–Schwarz domain decomposition algorithm. Our simulations show that coronary revascularization significantly improved myocardial perfusion, increasing the fractional flow reserve (FFR) from 0.742 to 0.904, but led to a 2.49% decrease in cerebral blood flow, potentially elevating the risk of cerebral ischemia. Strong parallel scalability was demonstrated on thousands of processor cores, highlighting the efficiency and applicability of domain decomposition methods for complex patient-specific blood flow simulations.

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Multiscale modeling of wave propagation with exponential integration and GMsFEM

Yin Yang

Xiangtan University

Numerical simulation of wave propagation in heterogeneous media is widely used in engineering and various applications. However, direct numerical simulation of wave propagation in such media is often impractical due to the need for extremely small time steps and fine spatial grids. In this talk, we introduce a new multiscale model reduction method for solving the wave equation in heterogeneous media by combining the generalized multiscale finite element method (GMsFEM) with exponential integrators (EI). For spatial discretization, we construct local multiscale basis functions within the GMsFEM framework to capture microscopic behavior. EI enhances time integration stability, overcoming limitations of traditional finite difference schemes for time discretizations in high-contrast media. Numerical examples are presented to demonstrate the efficiency of our method.

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Learning-based multilevel solvers for large-scale linear systems

Chensong Zhang

Academy of Mathematics and Systems Science

This talk presents our efforts on developing learning-based solvers for large-scale linear systems arising from discretized PDEs. Our approach bridges traditional multilevel solvers with machine learning, automating solver design to enhance efficiency and scalability. The method generalizes across grid sizes, coefficients, and right-hand-side terms, enabling offline training and efficient generalization, with convolutional neural networks (CNNs) serving as the basic computational kernels. It utilizes multilevel hierarchy for rapid convergence and cross-level weight sharing to adapt flexibly to varying grid sizes. The proposed solver achieves speedup over classical geometric multigrid methods for convection-diffusion PDEs in preliminary numerical experiments. We further extend this framework by introducing a Fourier neural network (FNN) to accelerate source influence propagation in Helmholtz equations within heterogeneous media. Supervised experiments demonstrate superior accuracy and efficiency compared to other neural operators, while unsupervised scalability tests reveal significant speedups over other AI solvers, achieving near-optimal convergence for wave numbers up to $k \approx 2000$. Ablation studies validate the effectiveness of the multigrid hierarchy and the novel FNN architecture.

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Two-level hybrid domain decomposition method for Helmholtz problem with high wave number

Bowen Zheng

Academy of Mathematics and Systems Science, Chinese Academy of Science

In this talk, we discuss and analyze two-level hybrid Schwarz preconditioners for solving the Helmholtz equation with high wave number in two and three dimensions. Both preconditioners are defined over a set of overlapping subdomains, with each preconditioner formed by a global coarse solver and one local solver on each subdomain. The global coarse solver is based on the localized orthogonal decomposition (LOD) technique, which was proposed originally for the discretization schemes for elliptic multiscale problems with heterogeneous and highly oscillating coefficients and Helmholtz problems with high wave number to eliminate the pollution effect. The local subproblems are Helmholtz problems in subdomains with homogeneous boundary conditions (the first preconditioner) or impedance boundary conditions (the second preconditioner). Both preconditioners are shown to be optimal under some reasonable conditions, that is, a uniform upper bound of the preconditioned operator norm and a uniform lower bound of the field of values are established in terms of all the key parameters, such as the fine mesh size, the coarse mesh size, the subdomain size and the wave numbers. It is the first time to show that the LOD solver can be a very effective coarse solver when it is used appropriately in the Schwarz method with multiple overlapping subdomains. Numerical experiments are presented to confirm the optimality and efficiency of the two proposed preconditioners. The global coarse solvers involved in the two preconditioners can be quite expensive for very large wave numbers. At the second part of the talk, we will explore the possibilities to reduce the complexity of the global coarse solvers. We construct a novel one-level additive DDM for solving the coarse problem in the two-level framework, and propose some propositions for its well-posedness, efficiency and convergence.

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MS12 – Numerical methods for time-harmonic wave propagation problems

Organizers: Pierre-Henri Cocquet, Martin J. Gander

Time-harmonic wave propagation problems are numerically challenging because they combine multiple difficulties like highly oscillatory solutions, complex non-Hermitian and indefinite discrete operators, and they are often posed on unbounded domains. In addition, most numerical methods suffer from dispersion error which is the fact that plane waves at the discrete level oscillate at a different frequency from the continuous one. The dispersion error is responsible for the so-called pollution effect indicating that having a fixed number of grid points per wavelength is not enough to prevent the relative error to grow with the wave number.

Due to all these difficulties, the development of numerical methods for time-harmonic wave propagation problems is currently a very active field of research. Reducing dispersion error or the pollution effect permits for instance to have, for a given mesh size, smaller relative error, and to enhance the convergence properties of certain iterative methods, like for example multigrid. New domain decomposition methods are also using as transmission conditions techniques which were developed for the truncation of unbounded domains, like absorbing boundary conditions, perfectly matched layers, the pole condition and so on.

The goal of this minisymposium is to gather researchers in numerical wave propagation in order to gain an overview of the most recent techniques and methods.

List of Speakers

- Y. Boubendir (New Jersey Institute of Technology) – *Improved domain decomposition methods using cross-points treatment for the Helmholtz equation*.
- P.-H. Cocquet (Université de Pau et des Pays de l'Adour, SIAME) – *Asymptotic dispersion correction for the Yee scheme applied to Maxwell's equations*.
- P. Marchner (Université Grenoble Alpes) – *Schwarz domain decomposition and domain truncation for exterior time-harmonic problems with variable coefficients and convective flows*.
- M. Michelle (Purdue University) – *Sixth-order compact finite difference method for 2D Helmholtz equations with singular sources and reduced pollution effect*.
- A. Modave (POEMS, CNRS, Inria, ENSTA, Institut Polytechnique de Paris) – *A numerical comparison of hybridization strategies for the iterative finite element solution of Helmholtz problems*.
- S. Pescuma (ENSTA) – *A HDG method with transmission variables for general time-harmonic wave problems: application to convected acoustics*.
- M. Rivet (EPCI Makutu, Inria, Université de Pau et des Pays de l'Adour, Université de Toulouse) – *A quasi-Trefftz domain decomposition method for the iterative solution of time-harmonic wave problems*.
- C. Stolk (University of Amsterdam) – *Dispersion correction for finite element Helmholtz equations on unstructured meshes*.

- D. Wörgötter (TU Wien) – *Wavenumber-explicit hp-FEM analysis of Maxwell's equations in piecewise smooth media.*
- H. Wu (Nanjing University) – *Dispersion analysis of CIP-FEM for Helmholtz equation.*
- Y. Yu (Guangxi University) – *Adaptive nonoverlapping preconditioners for the Helmholtz equation.*
- H. Zhang (Xi'an Jiaotong-Liverpool University) – *Compact high order schemes by interpolation with polynomials and plane waves.*

Improved domain decomposition methods using cross-points treatment for the Helmholtz equation

Yassine Boubendir

New Jersey Institute of Technology

In this talk we describe a procedure based on domain decomposition algorithms in order to derive an efficient iterative solver for the solution of the Helmholtz equation. We will explain several types of transmission conditions as well as the approach used to deal with the cross-points problem. We will show how to use this approach to further improve the convergence of the iterative procedure. Various numerical tests are presented.

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Asymptotic dispersion correction for the Yee scheme applied to Maxwell's equations

Pierre-Henri Cocquet

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This talk will focus on reducing the dispersion error of the Yee finite difference scheme for the time-harmonic Maxwell's equations. The proposed method, called asymptotic dispersion correction, has been developed for the Helmholtz equation and we are going to show how it can be extended to time-harmonic Maxwell equations. The latter is based on the introduction of a shifted angular frequency depending on a free parameter in the Yee stencil. The optimal parameter, called the asymptotically optimal shift, can then be determined explicitly by minimizing the dispersion error for a large enough number of grid points per wavelength. We will also present some numerical experiments to show that the relative error is reduced when using the optimal shifted angular frequency as soon as the number of grid point per wavelength is large enough.

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Schwarz domain decomposition and domain truncation for exterior time-harmonic problems with variable coefficients and convective flows

Philippe Marchner

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Solving time-harmonic wave problems with iterative methods is notoriously challenging at high wavenumbers. The numerical difficulties encountered for Helmholtz problems are intensified when waves propagate through heterogeneous media and under a background convective flow. Schwarz domain decomposition splits the computational domain into subdomains at the PDE level and can be used as a hybrid direct–iterative solver, enabling large-scale simulations on modern computing architectures up to thousands of subdomains. The convergence rate of Schwarz methods depends on the choice of transmission conditions, which ideally correspond to a nonlocal, hence computationally expensive Dirichlet-to-Neumann (DtN) operator on the complement of each subdomain. In this talk, I will present local approximations of DtN operators tailored to exterior, time-harmonic problems with spatially varying coefficients and flow convection. I will discuss several families of rational and polynomial approximations, and relate them to domain truncation techniques such as absorbing boundary conditions and perfectly matched layers. I will present simple numerical examples using high-order finite elements and a substructured, non-overlapping Schwarz formulation to illustrate successful applications, and highlight challenges associated with corners.

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Sixth-order compact finite difference method for 2D Helmholtz equations with singular sources and reduced pollution effect

Michelle Michelle

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The Helmholtz equation is numerically challenging to solve because its solution is highly oscillating. A mesh size that is smaller than the reciprocal of the wavenumber is often needed to produce a reasonable solution (a phenomenon known as the pollution effect). High-order schemes for solving the Helmholtz equation are very much desirable due to their ability in mitigating the pollution effect. In this talk, I will present a high-order compact finite difference method for 2D Helmholtz equations with singular sources and mixed boundary conditions on a rectangular domain. Our method is sixth-order and fifth-order consistent for constant and piecewise constant wavenumbers respectively. We employ a new pollution minimization strategy that is based on the average truncation error of plane waves. Some comparisons with existing finite difference methods will be discussed.

Joint work with Q. Feng and B. Han.

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A numerical comparison of hybridization strategies for the iterative finite element solution of Helmholtz problems

Axel Modave

POEMS, CNRS, Inria, ENSTA, Institut Polytechnique de Paris

We consider the iterative solution of large-scale Helmholtz problems discretized with finite element methods. These problems are notoriously difficult to solve iteratively because the matrices of the linear systems are generally large, sparse, complex and indefinite.

In this work, we are interested in the effect of the hybridization and static condensation strategies on the properties of the linear system. These strategies can be interpreted as substructuring techniques applied to high-order finite element schemes: they modify the properties of the linear system without changing the original solution, which can be recovered in a post-processing step.

We have recently proposed and studied hybridizable discontinuous Galerkin (HDG) methods with transmission variables, which perform better than the standard approaches for the iterative solution of Helmholtz problems, see [1,2]. In this talk, we will describe and compare these approaches by using several reference 2D numerical simulations and different iterative methods (fixed-point, CGNR and GMRES). A preliminary comparison with standard continuous Galerkin finite element methods will also be proposed.

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A HDG method with transmission variables for general time-harmonic wave problems: application to convected acoustics

Simone Pescuma

ENSTA

Discontinuous Galerkin (DG) finite element methods are effective for solving time-harmonic wave propagation problems, such as the Helmholtz equation, on complex domains due to their high-order accuracy on unstructured meshes. However, the resulting linear systems are often large and poorly conditioned, making direct solvers impractical and iterative methods slow, particularly at high frequencies.

To address these challenges, hybridizable DG methods (HDG) have been developed. Among them, a new approach (CHDG) has been introduced for the Helmholtz equation in homogeneous media. CHDG defines auxiliary unknowns using transmission variables at element faces, allowing for the elimination of internal fields and the formulation of a hybrid system. This approach can be interpreted as a non-overlapping Schwarz substructuring domain decomposition method applied element by element to an upwind DG scheme. The resulting hybrid system is suitable for efficient iterative solvers like CGNR, GMRES, and fixed-point methods, which outperform iterative solutions with standard DG and HDG approaches in terms of number of iterations.

This work extends the CHDG method to general symmetric hyperbolic systems with constant physical coefficients, focusing on wave propagation in the presence of a constant subsonic background flow, as modeled by the convected Helmholtz equation. The transmission variables correspond to the characteristic variables obtained in a Riemann solver. Under suitable hypothesis on the boundary conditions, solvability via fixed-point iteration is preserved.

Numerical tests on 2D benchmarks confirm the efficiency and robustness of the CHDG approach, which proves to be well-suited for iterative solution strategies in such contexts.

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A quasi-Trefftz domain decomposition method for the iterative solution of time-harmonic wave problems

Matthias Rivet

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The rise in the use of high-frequency electromagnetic waves calls for the numerical simulation of propagation phenomena in domains of growing dimension and complexity. Many numerical methods have been developed to address such problems, as the Finite Difference, Finite Element or Discontinuous Galerkin methods. Yet, considering the large domain dimensions of current time-harmonic frameworks (in terms of wavelengths), such methods lead to linear systems whose direct resolution is memory-prohibitive, while the associated matrices are poorly suited to iterative algorithms. On the contrary, Trefftz methods, which can be interpreted as a Discontinuous Galerkin formulation in which local basis functions are actual solutions of the considered PDE, are able to face such computational challenges thanks to pertinent properties ensuring their adaptability to iterative solution [1]. However, the classic choice of plane waves tends to induce ill-conditioned local systems, leading us to consider a quasi-Trefftz method: it relies on computed approximate Maxwell solutions, associated with piecewise polynomial boundary conditions, taken as new local basis functions [2]. In this talk, we propose a quasi-Trefftz approach based on a Flux Reconstruction [3] local solver: the basis quality and convergence properties (in terms of mesh and number of basis functions) of this method will be numerically investigated to demonstrate the robustness and precision of this methodology in an HPC context. Ultimately, in the framework of the iterative solution of the Trefftz formulation, the reduction of the number of iterations to convergence will be investigated through the introduction of new numerical fluxes. Interpreting this approach from a Domain Decomposition point of view, their definition will rely on the approximation of the exterior Dirichlet-to-Neumann operator thanks to classical analytic expressions and Neural Networks.

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Dispersion correction for finite element Helmholtz equations on unstructured meshes

Chris Stolk

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Dispersion errors are a large cause of errors in time-harmonic wave simulations. Correcting them can, on balance, reduce errors in numerical solutions as was shown in recent years for finite difference problems. Reducing dispersion errors can also greatly improve the convergence behavior of two-grid or multigrid methods (Stolk et al. 2014). In this talk we will discuss methods for dispersion correction for the Helmholtz equation discretized by continuous Galerkin methods on unstructured meshes.

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Wavenumber-explicit hp-FEM analysis of Maxwell's equations in piecewise smooth media

David Wörgötter

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We consider the high-frequency time-harmonic Maxwell equations with impedance boundary conditions on a domain with analytic boundary. We suppose that the considered domain consists of multiple subdomains and consider permeability and permittivity tensors that are analytic on every subdomain, but may jump across subdomain interfaces.

Under these conditions we show that for any wavenumber $k > 1$, a Galerkin discretization based on Nédélec-elements of order p on a mesh with mesh-width h is quasi-optimal, provided there holds the k-explicit scale resolution condition a) that kh/p is sufficiently small and b) that $p/\log k$ is bounded from below.

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Adaptive nonoverlapping preconditioners for the Helmholtz Equation

Yi Yu

Guangxi University

One of the issues with traditional preconditioning of the Helmholtz equations is the potential ill-posedness of the local Dirichlet boundary problem. In this talk, we introduce a new iterative substructuring method, which is similar in concept to the Schur complement system used for elliptic problems. This new structure ensures the well-posedness of the local Dirichlet problems by incorporating the small-magnitude eigenvalues from each subdomain into the coarse problem. Another key challenge of traditional preconditioning of the Helmholtz equations lies in constructing an effective coarse space for non-overlapping methods. Motivated by the success of using generalized eigenvalue problems to precondition elliptic equations with heterogeneous coefficients, we propose two types of DDMs that construct a robust coarse problem. Moreover, our construction is purely algebraic, facilitating straightforward extension to other discretizations and the case of heterogeneous Helmholtz coefficients. While the convergence theorems only hold when the thresholds are very close to zero, numerical performance of the method works well in an asymptotic sense.

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Dispersion analysis of CIP-FEM for Helmholtz equation

Haijun Wu

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When solving the Helmholtz equation numerically, the accuracy of numerical solution deteriorates as the wave number k increases, known as 'pollution effect' which is directly related to the phase difference between the exact and numerical solutions, caused by the numerical dispersion. In this paper, we propose a dispersion analysis for the continuous interior penalty finite element method (CIP-FEM) and derive an explicit formula of the penalty parameter for the p^{th} order CIP-FEM on tensor product (Cartesian) meshes, with which the phase difference is reduced from $\mathcal{O}\big(k(kh)^{2p}\big)$ to $\mathcal{O}\big(k(kh)^{2p+2}\big)$. Extensive numerical tests show that the pollution error of the CIP-FE solution is also reduced by two orders in kh with the same penalty parameter.

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Compact high order schemes by interpolation with polynomials and plane waves

Hui Zhang

Xi'an Jiaotong-Liverpool University

Babuska-Sauter-1997 derived a dispersion free scheme in 1D that is exact for the plane waves as well as cell-wise constant source terms. They also showed that in 2D the pollution effect is unavoidable. Babuska-Ihlenburg-Paik-Sauter-1995 derived a quasi-stabilized scheme in 2D that is exact for 16 plane waves with angles $\frac{\pi}{16} + j\frac{\pi}{8}$ for $j = 0, 1, .., 15$. It is interesting to generalize the idea to high order schemes and 3D. To this end, we propose an approach based on interpolation. We obtain 6th order schemes on the 9-point stencil in 2D and the 27-point stencil in 3D that are exact for selected sets of plane waves to minimize dispersion errors.

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MS13 – Machine learning-enhanced solvers: advances in domain decomposition and operator learning for PDEs

Organizers: Victorita Dolean-Maini, Marco Verani

This mini-symposium focuses on the integration of machine learning techniques with numerical solvers, with a particular emphasis on domain decomposition methods and their application to challenging problems in partial differential equations (PDEs). Domain decomposition has long been a cornerstone of scalable and efficient solvers, and the recent emergence of machine learning offers new opportunities to enhance its capabilities. The speakers will showcase innovative approaches that combine deep learning with classical numerical frameworks to address the computational challenges posed by high-dimensional, nonlinear, and mixed-dimensional systems. Key topics include:

Neural Operator-Based Solvers: Accelerating Newton's method for nonlinear elliptic PDEs using Fourier neural operators to improve efficiency and scalability. Learning-Driven Finite Element Methods: Enhancements to polytopal methods through machine learning, expanding their applicability and robustness. Domain Decomposition and Preconditioning: Machine learning-inspired preconditioners for random feature methods and domain decomposition approaches for both linear and nonlinear problems, including physics-informed neural networks (PINNs). Efficient Solutions for Helmholtz and Mixed-Dimensional Problems: Neural network-driven preconditioning techniques to tackle challenging equations like the Helmholtz equation and mixed-dimensional systems. DeepONet for Solvers: Novel uses of DeepONet for approximating inverse operators and their implications for efficient and scalable linear solvers.

This symposium emphasizes the potential of machine learning to enhance solver performance, particularly in the context of domain decomposition, which remains critical for solving large-scale and complex problems.

List of Speakers

- J. Aghili (Université de Strasbourg) – *Accelerating Newton's convergence for nonlinear elliptic PDEs using Fourier neural operators.*
- P. F. Antonietti (Politecnico di Milano) – *Machine-learning enhanced (polytopal) finite element methods.*
- M. Caldana (Politecnico di Milano) – *A deep learning algorithm to accelerate algebraic multigrid methods.*
- N. Dimola (Politecnico di Milano) – *Efficient solution of mixed-dimensional PDEs using a neural preconditioner.*
- A. Heinlein (Delft University of Technology) – *Domain decomposition for physics-informed neural networks: linear and nonlinear function approximation and operator learning.*
- D. Hrebenshchykova (Université Côte d'Azur) – *Neural network-driven domain decomposition for efficient solutions to the Helmholtz equation.*

- C. Millevoi (University of Padova) – *Learning low-frequency eigenmodes with DeepONet*.
- J. W. van Beek (TU Eindhoven) – *Preconditioning random feature methods*.

Accelerating Newton's method for nonlinear elliptic PDEs using Fourier neural operators

Joubine Aghili

IRMA, Université de Strasbourg

This talk will address a critical challenge in solving nonlinear elliptic partial differential equations (PDEs): the slow convergence of Newton's method when the initial guess is far from the true solution. To overcome this, we use Fourier Neural Operators (FNOs), a class of neural operators designed for learning mappings between function spaces, to generate high-quality initial guesses. FNOs are particularly appealing because they are resolution-invariant, meaning they can be trained on coarse grids and applied to finer ones without retraining, making them computationally efficient for PDE problems. The FNO is trained to predict an approximate solution to the discretized PDE by minimizing a loss function based on the PDE residual, using data generated from numerical simulations or analytical solutions. The training process involves creating a dataset of input-output pairs, where inputs are problem parameters (e.g., coefficients or boundary conditions) and outputs are corresponding PDE solutions. The FNO learns to map these parameters to approximate solutions, which serve as initial guesses for Newton's method. We show numerically that this approach significantly reduces the number of Newton iterations required for convergence, especially for strongly nonlinear or anisotropic PDEs, where traditional initial guesses (e.g., zero or linear interpolations) perform poorly. Numerical experiments in one and two dimensions validate the approach. For the anisotropic elliptic PDEs, the FNO-initialized Newton's method achieves convergence in fewer iterations compared to baseline methods, with minimal computational overhead from the FNO evaluation. We will also discuss the trade-offs, such as the cost of training the FNO and the need for sufficient training data to generalize across problem variations.

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Machine-learning enhanced (polytopal) finite element methods

Paola F. Antonietti

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In this talk, we discuss the integration Machine Learning (ML) techniques into (polytopal) Finite Element Methods to enhance their accuracy and flexibility while addressing the complexities encountered in practical applications such as computational neuroscience and geoscience. In the first part of the talk, we present innovative ML-driven mesh agglomeration strategies in polytopal Finite Element Methods. We introduce a novel algorithm based on Graph Neural Networks to process the connectivity graph of mesh elements and the physical properties of the model under analysis simultaneously, thereby agglomerating mesh elements and ensuring the generation of high-quality coarse grids. The resulting agglomerated meshes can be employed to reduce the computational burden while maintaining a detailed representation of the geometry and constructing efficient grid hierarchies for geometric Multigrid methods, demonstrating significant improvements in computational efficiency. In the second part of the talk, we present a novel method based on deep learning to accelerate the convergence of Algebraic Multigrid (AMG) techniques. Specifically, ANNs are trained to predict the optimal strong connection parameter that governs the sequence of coarsened matrix problems within the AMG algorithm, thereby effectively reducing the time to a solution. We examine diverse differential problems and discretisation schemes to validate the proposed methodologies, including Virtual Element Methods (VEMs) and polytopal Discontinuous Galerkin (PolyDG) methods. This highlights the potential of ML to advance numerical simulations in complex physical domains.

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A deep learning algorithm to accelerate algebraic multigrid methods

Matteo Caldana

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Algebraic multigrid (AMG) methods are among the most efficient solvers for linear systems of equations and they are widely used for the solution of problems stemming from the discretization of Partial Differential Equations (PDEs). A severe limitation of AMG methods is the dependence on parameters that require to be fine-tuned. In particular, the strong threshold parameter is among the most relevant parameter since it stands at the basis of the construction of successively coarser grids needed by the AMG methods. We present a novel deep learning algorithm to accelerate the convergence of AMG methods. An artificial neural network, comprised of a dense and convolutional part, tunes the value of the strong threshold parameter by interpreting the sparse matrix of the linear system as a gray-scale image and exploiting a pooling operator to transform it into a small multi-channel image. To demonstrate the practical capabilities of the proposed algorithm, we apply it to the iterative solution of linear systems of equations stemming from finite element discretizations of two- and three-dimensional model problems with structured, unstructured, and polytopal grids. Namely, we consider diffusion equations with a highly heterogeneous coefficient, stationary Stokes problems, and linear elasticity problems with a highly heterogeneous Young's modulus. When tested on problems with coefficients or geometries not present in the training dataset, our approach reduces the computational time by up to 30% with respect to using the value commonly found in the literature.

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Efficient solution of mixed-dimensional PDEs using a neural preconditioner

Nunzio Dimola

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Mixed-dimensional partial differential equations (PDEs) are characterized by coupled operators defined on domains of varying dimensions and pose significant computational challenges due to their inherent ill-conditioning. Moreover, the computational workload rises considerably when attempting to accurately capture the behavior of the system under significant variations or uncertainties in the low-dimensional structures such as fractures, fibers, or vascular networks, due to the inevitable necessity of running multiple simulations. In this work, we present a novel preconditioning strategy that leverages neural networks and unsupervised operator learning to design an efficient preconditioner specifically tailored to a class of 3D-1D mixed-dimensional PDEs. The proposed approach is capable of generalizing to varying shapes of the 1D manifold without retraining, making it robust to changes in the 1D graph topology. Moreover, thanks to convolutional neural networks, the neural preconditioner can adapt over a range of increasing mesh resolutions of the discrete problem, enabling us to train it on low resolution problems and deploy it on higher resolutions. Numerical experiments validate the effectiveness of the preconditioner in accelerating convergence in iterative solvers, demonstrating its appeal and limitations over traditional methods. This study lays the groundwork for applying neural network-based preconditioning techniques to a broader range of coupled multi-physics systems.

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Domain decomposition for physics-informed neural networks: linear and nonlinear function approximation and operator learning

Alexander Heinlein

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Physics-Informed Neural Networks (PINNs) offer a flexible, mesh-free approach to solving differential equations using neural networks. While their implementation is straightforward, training remains challenging due to sensitivity to weight initialization and hyper parameter selection. Additionally, issues such as scalability, spectral bias, and ill-conditioning pose significant challenges. This talk explores how overlapping domain decomposition (DD) techniques can improve the convergence and efficiency of PINNs.

Three primary cases are investigated. First, classical PINNs are discussed, where solutions to specific initial-boundary value problems are approximated using standard feed-forward neural networks. To enhance performance, DD-based architectures are introduced. Second, randomized neural networks are considered, where hidden layer weights are randomly initialized and fixed, leaving only the final layer to be trained. For linear differential operators, this approach reduces the problem to a linear least-squares formulation, which can be efficiently solved using classical numerical techniques. In this context, DD-based architectures are implemented alongside overlapping Schwarz preconditioning to accelerate convergence. Finally, improvements in physics-informed neural operators through DD-based architectures are studied. Unlike solving a single initial-boundary value problem, neural operators approximate solution operators for parameterized problems.

Numerical experiments conducted on various model problems, including multiscale and wave phenomena, demonstrate the effectiveness of domain decomposition techniques for enhancing PINNs. These results highlight the potential of DD methods to improve computational efficiency and accuracy in physics-informed machine learning.

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Neural network-driven domain decomposition for efficient solutions to the Helmholtz equation

Daria Hrebenshchykova

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Accurately simulating wave propagation is crucial in fields such as acoustics, electromagnetism, and seismic analysis. Traditional numerical methods, like finite difference and finite element approaches, are widely used to solve governing partial differential equations (PDEs) such as the Helmholtz equation. However, these methods face significant computational challenges when applied to high-frequency wave problems in complex two-dimensional domains.

This work investigates Finite Basis Physics-Informed Neural Networks (FBPINNs) and their multilevel extensions as a promising alternative. These methods leverage domain decomposition, partitioning the computational domain into overlapping subdomains, each governed by a local neural network. We assess their accuracy and computational efficiency in solving the Helmholtz equation for the homogeneous case, demonstrating their potential to mitigate the limitations of traditional approaches.

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Learning low-frequency eigenmodes with DeepONet

Caterina Millevoi

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Preconditioners play a crucial role in efficiently solving linear systems arising from PDE discretization. Traditional single-level methods, such as Jacobi, Incomplete LU (ILU) factorization, and Factorized Sparse Approximate Inverse (FSAI), effectively reduce high-frequency error components but struggle with low-frequency components. Multigrid and two-level domain decomposition methods address this issue by balancing coarse-grid correction with smoothing. In this work, we introduce a novel approach for approximating the inverse of discrete operators using DeepONet, a supervised learning framework for nonlinear operators. Since this deep learning-based method is particularly well-suited for capturing low-frequency features, we construct a structure analogous to a conventional coarse grid in the multigrid sense by training DeepONet on vectors representing low-frequency components. As an alternative to classical single-level preconditioners, our DeepONet-based approximation effectively compensates for the low-frequency part of the eigenspectrum, accelerating convergence. We present preliminary test cases to assess the potential of this approach, with a view toward its application in matrix-free multi-level methods.

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Preconditioning random feature methods

Jan Willem van Beek

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In recent years, machine learning has emerged as a powerful tool for solving partial differential equations (PDEs), with randomized neural networks showing remarkable potential. These networks, typically shallow, differ from conventional approaches by randomizing all parameters except those in the final layer. Notably, by integrating techniques inspired by domain decomposition, they achieve improved local accuracy and computational efficiency. We refer to these approaches as random feature methods. Despite their promise, random feature methods lead to highly ill-conditioned least squares systems, posing a significant computational challenge. To address this, we introduce a novel approach based on local rank-revealing QR factorization. This technique provides two key advantages: first, it enables the selective elimination of nonexpressive features, reducing system size and improving the condition number without sacrificing accuracy; second, it facilitates the construction of an efficient preconditioner for the global system. Our numerical experiments demonstrate that this strategy can rival or even outperform existing preconditioning techniques, both in terms of condition number reduction and the efficiency of iterative solvers. These results underscore the potential of our method as a powerful enhancement to machine learning-based PDE solvers.

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MS14 – Continuous and discrete approaches to DDM: solvers and preconditioners with applications

Organizers: Pablo Jose Lucero Lorca, Connor McCoid, Michal Outrata

Schwarz methods are usually introduced and studied in a continuous setting, but their use is almost exclusively as algebraic solvers or preconditioners for discretized problems. Discrete concepts such as Jacobi and Gauss-Seidel preconditioners, Schur complements, and M-matrices have continuous counterparts in Schwarz methods, static condensation, and spectral decompositions. In particular, for any purely continuous formulation of a Schwarz method, there exists its discrete analogue. This analogue may have been explored using qualitatively different tools. This minisymposium collects talks on some of the latest advances in both continuous and discrete settings with the goal of exposing experts to new perspectives and deepening understanding of domain decomposition methods.

List of Speakers

- H. Chu (University of Pavia) – *Multigrid algorithm for immersed finite element discretizations of elliptic interface problems*.
- T. Hammerbauer (Charles University) – *Hybrid Schwarz preconditioners for linear systems arising from hp-discontinuous Galerkin method*.
- J. P. Lucero Lorca (–) – *On the relation between some direct and iterative multilevel solvers*.
- B. Mann (Friedrich-Alexander-Universität Erlangen-Nürnberg) – *Adaptive mesh refinement on hierarchical hybrid grids*.
- P. Marchand (Inria) – *Overlapping Schwarz preconditioner with GenEO coarse space for non-local equations*.
- C. McCoid (McMaster University) – *Symmetrized cells in adaptive optimized Schwarz*.
- M. Outrata (Charles University) – *Domain truncation, Schur complements, and Padé approximation*.
- N. Tian (Institute of Applied Physics and Computational Mathematics, Beijing, China) – *Mixed precision block-Jacobi preconditioner and its application in radiation hydrodynamics problems*.

Multigrid algorithm for immersed finite element discretizations of elliptic interface problems

Hanyu Chu

University of Pavia

This talk is devoted to analyzing multigrid algorithm for solving elliptic interface problems discretized using the partially penalized immersed finite element (PPIFE). By taking the average values of nodal variables and integral variables, we construct intergrid transfer operators for the P_1 partially penalized immersed finite element (P_1-PPIFE) and the Crouzeix-Raviart partially penalized immersed finite element (CR-PPIFE), which satisfy certain stable approximation properties. An extra interface correction procedure is added in the smoothing steps to ensure the robustness of multigrid algorithm. We prove that the convergence of W-cycle multigrid algorithm and the condition number of the variable V-cycle as preconditioner are optimal by verifying the regularity-approximation assumption, which means that the convergence rate of algorithm is independent of mesh level, mesh size, and the position of the interface relative to the mesh. Numerical experiments illustrate the convergence of our algorithms using the W-cycle, V-cycle, and preconditioned conjugate gradient algorithm (PCG) with the V-cycle.

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Hybrid Schwarz preconditioners for linear systems arising from hp-discontinuous Galerkin method

Tomas Hammerbauer

Charles University

We present the analysis and numerical study of two-level hybrid Schwarz method used as a preconditioner for system of algebraic equations arising from discontinuous Galerkin (DG) discretization. The preconditioner is additive with respect to the local components and multiplicative with respect to the mesh levels. We present theoretical results concerning hp condition number bounds of the preconditioned system. Moreover, we show the numerical results concerning the computation of those bounds to demonstrate its accuracy. Finally, we present results showing the dominance of this approach in comparison to the additive Schwarz method from the point of view of the speed of convergence and also computational costs.

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On the relation between some direct and iterative multilevel solvers

Jose Pablo Lucero Lorca

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In a DD26 proceeding, we reported the appearance of clustered eigenvalues when optimizing the restriction and prolongation operators of a two-level method using 1D Local Fourier Analysis (LFA) of the full two-level operator. This led to an iterative solver that converges to machine precision within a small, fixed number of iterations. Further investigation revealed an explanation linking LFA to cyclic reduction interpreted as a multigrid method. In this talk, we present the details of those 1D findings, along with a 2D connection between the Hierarchical Poincaré-Steklov method—a nested dissection solver for spectral element discretizations—and its interpretation as an algebraic multigrid method with block-Jacobi smoothers, in which geometric information is retained during coarsening. As a result, the method lies between geometric and algebraic multigrid approaches. We illustrate this connection with examples.

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Adaptive mesh refinement on hierarchical hybrid grids

Benjamin Mann

Friedrich-Alexander-Universität Erlangen-Nürnberg

Numerical simulations of extreme-scale systems can result in linear systems with trillions of unknowns. For instance, Earth mantle convection, modeled by the Stokes equations with a surface resolution of 1 km, presents such a challenge. Our finite element framework, HyTeG (Hybrid Tetrahedral Grids), offers efficient and flexible data structures alongside highly scalable algorithms to address a wide range of problems, including curl-curl, convection-diffusion, and Stokes equations. By leveraging block-structured grids, highly efficient matrix-free compute kernels can be generated automatically. In addition, such a structure also provides an inherent grid hierarchy, that can be utilized for geometric multigrid solvers. On the other hand, in contrast to fully unstructured grids, there are additional challenges when dealing with heterogeneous problems that require adaptive mesh refinement (AMR). The standard solution is to use different grid levels on each block, but this approach diminishes the performance benefits of the local grid structure. In this work, we propose an alternative: Instead of altering the level of structuredness, mesh adaptivity is applied exclusively to the coarse grid. Numerical experiments show that this method achieves convergence rates comparable to fully adaptive schemes, even in the presence of singularities. Apart from suitable refinement strategies, a posteriori error estimation is a crucial component of AMR. Although a wide range of sophisticated methods exist for estimating local and global errors, they tend to be computationally expensive. In this work, we employ an exceptionally cost-efficient estimator: The solution on the finest grid in the hierarchy serves as a proxy for the true solution to estimate the error on the coarser levels. This approach is particularly advantageous when combined with full multigrid (FMG) methods, where computing solutions on all grid levels is an inherent part of the solver, which means that the estimate is essentially free. We provide analytical bounds on the effectivity indices of the estimate, which are verified by numerical studies.

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Overlapping Schwarz preconditioner with GenEO coarse space for non-local equations

Pierre Marchand

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Domain Decomposition Methods, such as Additive Schwarz, can be used to precondition linear systems, and they usually rely on an additional coarse space to scale with the number of subdomains. The Generalized Eigenproblems in the Overlaps (GenEO) has emerged as one of the most promising coarse space for sparse symmetric positive definite problems, see [2]. GenEO takes eigenvectors of well-chosen local eigenproblems as a basis for the coarse space. As one of its interesting features, GenEO is only based on the knowledge of the stiffness matrix elements and discretization agnostic, left apart a few reasonable assumptions.

Recently, the GenEO approach has been extended to Boundary Integral Equations (BIEs) for the hypersingular operator in [1]. In this context, the discretized operator is non-local so that the resulting linear system is dense. Thus, the local eigenproblems used to build the GenEO coarse space are adapted to the non-local nature of the problem and its $\|\cdot\|_{\widetilde{H}^{1/2}}$ energy norm.

In this talk, we will present theoretical and numerical results aiming at adapting GenEO to the integral fractional Laplacian of order s for $0 < s < 1$. It shares many similarities with BIEs, e.g. its non-local nature and the energy norm $\|\cdot\|_{\widetilde{H}^{s}}$, that will be used to introduce a new distributed solver using the libraries PyNucleus, Htool-DDM and HPDDM.

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Symmetrized cells in adaptive optimized Schwarz

Conor McCoid

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Adaptive optimized Schwarz methods update transmission conditions at each iteration to achieve superlinear convergence. For multiple subdomains, this can be done by considering each subdomain individually, forming symmetrized cells, building transmission conditions for each cell, and then reconstructing a fast global Schwarz method. The process of building transmission conditions for the symmetrized cells has continuous analogues in methods for perfectly matched layers. This talk explores implementation options as well as issues such as crosspoints and scaling.

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Domain truncation, Schur Complements, and Padé approximation

Michal Outrata

Charles University

In this talk, we combine two classical tools – absorbing boundary conditions (ABCs) and continued fractions – and show for a model problem that the truncation of an unbounded domain by an artificial Dirichlet boundary condition placed far away from the domain of interest is equivalent to a specific absorbing boundary condition at the boundary of the domain of interest. We prove that the absorbing boundary condition thus obtained is a spectral Padé approximation about infinity of the transparent boundary condition. We also propose two additional ABCs motivated by this result and numerically show their efficiency for our test problem.

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Mixed precision block-Jacobi preconditioner and its application in radiation hydrodynamics problems

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This talk introduces our recent progress on mixed-precision Block-Jacobi preconditioners. We propose two mixed-precision Block-Jacobi preconditioners and investigate their performance through large-scale sparse linear systems arising from two classes of radiative hydrodynamics problems: radiation diffusion and radiation transport. We analyze the impact of mixed-precision Block-Jacobi preconditioners on the convergence of Krylov subspace methods. Some enlightening conclusions are obtained which are valuable for the design of more efficient mixed-precision preconditioners in solving practical applications.

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MS15 – Efficient numerical methods with machine learning for PDEs with singularities

Organizers: Alexander Heinlein, Xuejun Xu, Jun Zou

Singularities in partial differential equations (PDEs) present significant challenges in numerical analysis, often resulting from features such as point sources, discontinuities, transitions between distinct boundary conditions, or complex computational geometries. These phenomena reduce the regularity of solutions, requiring the careful selection of appropriate functional spaces and the design of specialized discretization and solution techniques—such as localized mesh refinement near singularities. This mini-symposium brings together researchers and practitioners to explore how machine learning can enhance the robustness and efficiency of classical numerical solvers for PDEs with singularities and how these solvers can, in turn, improve machine learning models.

Contributions are invited on innovative approaches, such as employing neural networks for discretization within traditional solvers like domain decomposition methods, or leveraging numerical methods to refine neural network architectures in approximating PDE solutions with singular behavior. Additionally, discussions on transmission and coupled problems are particularly welcome, as these cases pose severe challenges for both numerical methods and machine learning models. By highlighting promising methodologies as well as the potential challenges and limitations of using machine learning in this context, we aim at a diverse discussion about the interplay between machine learning and numerical techniques for PDEs with singularities.

List of Speakers

- A. Angino (Unidistance Suisse) – *Data-magical trust region: a multi-fidelity optimization strategy for neural network training*.
- E. Chung (The Chinese University of Hong Kong) – *Learning a generalized multiscale prolongation operator*.
- S. Gong (The Chinese University of Hong Kong) – *High order methods and robust block preconditioners for the fluid-rigid body interaction with applications in microfluidics*.
- H. H. Kim (Kyung Hee University) – *Neural network tearing and interconnecting methods to partial differential equations*.
- C.-O. Lee (KAIST) – *Learning axial Green's function via neural networks for elliptic problems with variable coefficients*.
- L. Luo (University of Macau) – *Domain decomposition preconditioned inexact Newton method with learning capability*.
- B. Ryoo (KAIST) – *A Neumann-Neumann acceleration with coarse space for domain decomposition of extreme learning machines*.
- X. Xu (Tongji University) – *Domain decomposition learning methods*.
- Y. Xu (Northeast Normal University) – *BPINN: a geometric structure-preserving physics-informed neural network for evolution problem with blow-up solutions*.

- Y. Yang (Guilin University of Electronic Technology) – *Machine learning accelerated solution of PNP equations with applications to ion channels*.
- S. Zhang (Chinese Academy of Sciences) – *A natural deep Ritz method for essential boundary value problems*.

Data-magical trust region: a multi-fidelity optimization strategy for neural network training

Andrea Angino

Unidistance Suisse

In recent years, multi-fidelity optimization has gained significant attention, particularly in areas characterized by complex and expensive objectives, such as neural network training. This work presents a novel adaptation of the Magical Trust Region (MTR) method, termed Data-Magical Trust Region (DMTR), which can be applied broadly in settings where a dataset is available and feature extraction is feasible; neural network training is considered as one representative application. The distinctive feature of this approach lies in a secondary search direction that exploits a computationally efficient surrogate of the full objective function. Unlike methods that rely solely on mini-batch evaluations of the loss, this direction is constructed through a feature extraction process applied to the dataset, which serves as the basis for the surrogate model. This enables the algorithm to capture relevant structural information cost-efficiently. As a result, the method enhances both the adaptability and the effectiveness of the optimization process during classifier training. We demonstrate the performance of DMTR across several datasets, highlighting consistent improvements over standard optimization strategies.

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Learning a generalized multiscale prolongation operator

Eric Chung

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Multigrid preconditioners are one of the most powerful techniques for solving large sparse linear systems. In this talk, we present Darcy flow problems with random permeability using iterative solvers, enhanced by a two-grid preconditioner based on a generalized multiscale prolongation operator, which has been demonstrated to be stable for high contrast profiles. To circumvent the need for repeatedly solving spectral problems with varying coefficients, we harness deep learning techniques to expedite the construction of the generalized multiscale prolongation operator. Considering linear transformations on multiscale basis have no impact on the performance of the preconditioner, we devise a loss function by the coefficient-based distance between subspaces instead of the plain L2-norm of the difference of the corresponding multiscale bases. We discover that leveraging the inherent symmetry in the local spectral problem can effectively accelerate the neural network training process. In scenarios where training data are limited, we utilize the Karhunen–Loeve expansion to augment the dataset. Extensive numerical experiments with various types of random coefficient models are exhibited, showing that the proposed method can significantly reduce the time required to generate the prolongation operator while maintaining the original efficiency of the two-grid preconditioner. Notably, the neural network demonstrates strong generalization capabilities, as evidenced by its satisfactory performance on unseen random permeability fields.

The research is partially supported by the Hong Kong RGC General Research Fund (Projects: 14305423 and 14305222).

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High order methods and robust block preconditioners for the fluid-rigid body interaction with applications in microfluidics

Shihua Gong

The Chinese University of Hong Kong, Shenzhen

We consider numerical algorithms for particulate flows, which involve the interactions between rigid particles and incompressible flows. In this talk, we will discuss computational challenges for this coupled system, including handling the moving interface between circular particles and flows, and addressing bottlenecks in large-scale and long-duration simulations. Our algorithms use arbitrary Eulerian-Lagrangian mapping to track the moving interface and distributed Lagrange multipliers to enforce the rigid-body motion. The advantages of our algorithms include maintaining the interface conditions exactly and featuring in high-order accuracy in both of the spatial and temporary discretization. At the end of the talk, we will discuss the stability analysis of the variational problems, which leads to robust parallel solvers for the linearized systems. We will also present some simulation results of particulate flows in microfluidics.

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Neural network tearing and interconnecting methods to partial differential equations

Hyea Hyun Kim

Kyung Hee University

A novel iterative algorithm is proposed for neural network approximation to partial differential equations, where partitioned neural networks are employed to approximate local solutions and the interface solution value is updated by using the neural network solutions. The interface solution update formula in the iterative algorithm is derived from a constrained energy minimization problem of the classical FETI formulation. The convergence of the iterative algorithm can be shown for a sufficiently small step size assumption on the update formula. However, the use of the small step size will slow down the iteration convergence. In order to speed up the iteration convergence, several preconditioning schemes are developed by using classical domain decomposition methods. The performance of the proposed algorithms is tested for various test examples including a discontinuous coefficient model problem.

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Learning axial Green's function via neural networks for elliptic problems with variable coefficients

Chang-Ock Lee

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The Axial Green Function Method (AGM) is a powerful computational framework for solving multidimensional elliptic boundary value problems by systematically decomposing them into a set of one-dimensional problems. While AGM avoids the complexity of full-dimensional Green's functions, its applicability is limited when analytic forms of the 1D Green's functions are unavailable due to variable coefficients. In this talk, we propose a neural network-based approach to overcome this limitation by learning the one-dimensional Green's function within the AGM framework. The neural Green's function is designed to satisfy homogeneous Dirichlet boundary conditions and the differential operator in a weak sense. Specifically, we represent the Green's function as a hybrid formulation comprising an analytically defined singular kernel and an MLP-based residual component. The neural network learns the smooth non-singular part, while singular structures are handled explicitly through classical Poisson kernels. We demonstrate the effectiveness of the method on benchmark elliptic problems with variable coefficients, and confirm that the structure and convergence properties of the original AGM are preserved. This hybrid approach integrates data-driven modelling with analytical numerical methods, and provides a more flexible and scalable extension of the Axial Green Function Method.

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Domain decomposition preconditioned inexact Newton method with learning capability

Li Luo

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Nonlinear preconditioning is a powerful technique that speeds up the convergence of nonlinear iterative methods for large, sparse nonlinear equations, especially for problems with local singularities. We present preconditioned inexact Newton methods with learning capability based on domain decomposition and neural networks. The proposed method learns the behavior of a Newton solver in the nonlinear residual and error space from a training problem, and incorporates neural network prediction in the preconditioning step to provide a better correction of the Newton iterate. Numerical experiments for high Reynolds number incompressible flow problems show that the proposed method is more robust and efficient compared with existing nonlinear solvers.

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A Neumann-Neumann acceleration with coarse space for domain decomposition of extreme learning machines

Byungeun Ryoo

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Extreme learning machines (ELMs), which preset hidden layer parameters and solve for last layer coefficients via a least squares method, can typically solve partial differential equations faster and more accurately than Physics Informed Neural Networks. However, they remain computationally expensive when high accuracy requires large least squares problems to be solved. Domain decomposition methods (DDMs) for ELMs have allowed parallel computation to reduce training times of large systems. This paper constructs a coarse space for ELMs, which enables further acceleration of their training. By partitioning interface variables into coarse and non-coarse variables, selective elimination introduces a Schur complement system on the non-coarse variables with the coarse problem embedded. Key to the performance of the proposed method is a Neumann-Neumann acceleration that utilizes the coarse space. Numerical experiments demonstrate significant speedup compared to a previous DDM method for ELMs.

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Domain decomposition learning methods

Xuejun Xu

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With the aid of hardware and software developments, there has been a surge of interest in solving partial differential equations by deep learning techniques, and the integration with domain decomposition strategies has recently attracted considerable attention due to its enhanced representation and parallelization capacity of the network solution. In this talk, a novel learning approach, i.e., the compensated deep Ritz method, is proposed to enable the flux transmission across subregion interfaces with guaranteed accuracy, thereby allowing us to construct effective learning algorithms for realizing the more general non-overlapping domain decomposition methods in the presence of overfitted interface conditions. Numerical experiments on a series of elliptic boundary value problems including the regular and irregular interfaces, low and high dimensions, smooth and high-contrast coefficients on multidomains are carried out to validate the effectiveness of our proposed domain decomposition learning algorithms.

Joint work with Q. Sun and H. Yi.

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BPINN: a geometric structure-preserving physics-informed neural network for evolution problem with blow-up solutions

Yingxiang Xu

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Physics-Informed Neural Networks (PINNs) have demonstrated remarkable success in solving partial differential equations, but struggle to solve evolution problems with blow-up solutions due to numerical instabilities near singularity. Based on the B-method proposed by Beck et al., we propose a geometric structure-preserving BPINN framework, which allows for a more accurate solving near the blow-up time, and more importantly, together with a novel adaptive time reduction strategy, it allows us to predict the blow-up time efficiently. The generalization error analysis is established. Several numerical examples are conducted to illustrate our results.

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Machine learning accelerated solution of PNP equations with applications to ion channels

Ying Yang

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The Poisson-Nernst-Planck (PNP) equations are a set of nonlinear coupled partial differential equations, which are widely used to describe the transport of charged particles in biological ion channels, electrochemical systems and semiconductors, etc. The PNP equations in biological ion channels present challenges for numerical algorithms due to their highly irregular geometric interfaces, multiple singularities, and nonlinear coupling. Numerical methods, such as the finite element method, are often combined with the Gummel iteration when solving the PNP equations. This iterative method is commonly used to decouple and linearize the PNP equations, but its efficiency is largely influenced by the relaxation parameter. The choice of the relaxation parameter is typically based on empirical selection, but this approach often fails to yield optimal values for practical problems, resulting in low efficiency. We apply two machine learning algorithms to predict the optimal parameters for the Gummel iteration, improving the efficiency of the iteration process. Specifically, we also apply these machine learning algorithms to solve practical PNP problems in various biological ion channels. Numerical experiments show that, compared to those selected by human experience, the parameters chosen by the machine learning algorithm are more stable and yield better results.

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A natural deep Ritz method for essential boundary value problems

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Implemented for solving elliptic partial differential equations, the deep Ritz method generally does not need the solution bear higher regularities than its variational formulation on proper Sobolev spaces. A main issue, however, lies on the treatment of the essential boundary value conditions, from which the robustness of the method often suffers. We introduce a parameter-free strategy to cope with the essential boundary condition by decomposing the original problem to natural boundary value problems which are still of elliptic type and which are easier to be solved by the deep Ritz method. The mathematical equivalence is proved rigorously based on the underlying structures. Numerical experiments illustrate the improvement of the robustness.

Joint work with H. Yu.

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MS16 – Robust and scalable algorithms for problems beyond benchmarking in computational science and engineering

Organizers: Luca Pavarino, Rongliang Chen, Li Luo

The increasing complexity of scientific and engineering problems demands the development of robust and efficient algorithms that go beyond traditional benchmarks. This mini-symposium focuses on cutting-edge methodologies addressing large-scale, nonlinear, and multi-physics challenges in fields such as computational fluid dynamics, biomechanics, material science, and geophysics. Topics of interest include advanced numerical schemes, scalable iterative solvers, preconditioning techniques, and machine-learning-enhanced computational frameworks. Special emphasis is placed on solutions tailored for real-world applications where conventional approaches may fail due to issues like extreme heterogeneity, high computational cost, or high accuracy requirements. Contributions demonstrating innovative algorithmic designs and their successful application to complex, real-world problems are encouraged. The goal is to foster interdisciplinary collaboration and inspire novel approaches that extend the frontiers of computational science and engineering.

List of Speakers

- P. Brubeck (University of Oxford) – *Large-scale simulations using classical and modern macroelements with Firedrake*.
- O. Farle (Dassault Systèmes) – *S-parameter computation for coupled meshes in a hybrid solver framework*.
- S. Gong (The Chinese University of Hong Kong) – *Power contractivity for RAS-Imp and RAS-PML for the time-harmonic wave equations*.
- C. Guillet (LIP6) – *High-performance isogeometric analysis of lattice structures*.
- J. Huang (Chinese Academy of Sciences) – *Derivatives of tree tensor networks and its applications in Runge–Kutta methods*.
- N. M. M. Huynh (Università degli Studi di Pavia) – *Novel scalable multilevel solvers for cardiac cell-by-cell model*.
- Y. Jiang (Shenzhen Institutes of Advanced Technology) – *A highly parallel domain decomposition algorithm for cardiac electro-mechanics simulations*.
- P. Lin (University of Dundee) – *A fast front-tracking approach for a temporal multiscale blood flow problem with a fractional boundary growth*.
- K. Nakajima (The University of Tokyo/RIKEN) – *Road to "AI for Science": exploring software sustainability through "Couplers"*.
- H. Yu (Academy of Mathematics and Systems Science, China) – *A low-diffusion positivity-preserving scheme with domain decomposition for high-Mach-number compressible turbulence*.

- S. Zampini (KAUST) – *Robust and scalable nonlinear solvers for finite element discretizations of biological transportation networks.*
- H. Zhang (Tsinghua University) – *An efficient Newton-Krylov method with domain decomposition for multi-physics coupling simulations in nuclear reactor power plants.*

Large-scale simulations using classical and modern macroelements with Firedrake

Pablo Brubeck

University of Oxford

Many classical and modern finite element spaces are derived by dividing each computational cell into finer pieces. Such macroelements frequently enable the enforcement of mathematically desirable properties such as divergence-free conditions or C1 continuity in a simpler or more efficient manner than elements without the subdivision. Although a few modern software projects provide one-off support for particular macroelements, a general approach facilitating broad-based support has, until now, been lacking. In this work, we describe a major addition to the Firedrake project to support a wide range of different macroelements. These enhancements have been integrated into the Firedrake code stack to enable high-productivity large-scale simulations in very few lines of code. We illustrate the approach with the numerical solution of various formulations of incompressible flow.

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S-parameter computation for coupled meshes in a hybrid solver framework

Ortwin Farle

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Multi-scale challenges frequently arise in the industrial applications of electromagnetic field simulation. One prominent example is determining the placement of antennas or antenna arrays on platforms such as cars, airplanes, or ships. Antennas often exhibit high geometric complexity and varying material properties, while platforms are typically electrically large. Accurate modeling of electromagnetic interference does not require detailed simulations of the platform, and performing a monolithic simulation of the entire system—including the antennas—is often infeasible.

Hybridization offers an effective solution to this problem. It involves simulating antennas and platforms using specialized solution methods tailored to the specific requirements of the task. For instance, the Hybrid Solver Task in CST Studio Suite® supports the integration of multiple techniques, such as the finite integration technique (FIT), finite element method (FEM), transmission line method (TLM), boundary element method (BEM), and beam-based asymptotic methods, all within an overlapping domain decomposition framework. These subdomains and solution methods are interconnected via field exchange on coupling surfaces.

Efficient computation of scattering parameters is essential in many applications. Reaction integrals on coupling surfaces provide an effective means of calculating these parameters, especially during the iteration on the surface unknowns. Electric and magnetic field strengths from different domains are required to calculate the reaction integrals. In addition to the mesh transfer of these field strengths, the presentation will also investigate the calculation of the magnetic field strengths from an electric field finite element formulation and its effect on the scattering parameters.

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Power contractivity for RAS-Imp and RAS-PML for the time-harmonic wave equations

Shihua Gong

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We consider two variants of restricted overlapping Schwarz methods for the Helmholtz equation and time-harmonic Maxwell equation. The first method, known as RAS-Imp, incorporates impedance boundary condition to formulate the local problems. The second method, RAS-PML, employs local perfectly matched layers (PML). These methods combine the local solutions additively with a partition of unity. We have shown that RAS-Imp has power contractivity for strip domain decompositions, and RAS-PML has super-algebraic convergence as wave number increases. In this talk we review these theoretical results and then present a massively parallel solver (with solving time linearly increasing w.r.t wave number) for Helmholtz. We also present some theoretical and numerical results of RAS-Imp for the time-harmonic Maxwell equation.

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High-performance isogeometric analysis of lattice structures

Clément Guillet

LIP6

Lattice structures are cellular architected materials that provide, among others, exceptional weight reduction while preserving stiffness and strength. Traditional multiscale methods based on homogenization are often inadequate for lattice structures due to insufficient scale separation. On the contrary, we focus here on high-fidelity, fine-scale simulations using isogeometric (spline-based) volumetric models at the architectural scale. The main challenge lies in the prohibitive computational cost of simulating the vast number of complex cells. Although isogeometric analysis (IGA) provides superior accuracy per degree of freedom compared to traditional finite element methods (FEM), it poses significant computational challenges, particularly in forming operators and solving the resulting linear systems. In this work, we introduce a high-performance solver tailored for the IGA of lattice structures, designed to harness the inherent characteristics of lattices (periodicity, multiscale) to overcome these computational barriers. The solver employs a two-level geometric preconditioner, combining a fine-level smoother based on overlapping domain decomposition with a coarse- level correction using an algebraic multigrid method. By leveraging the multiscale properties of lattice structures, the fine-level computations utilize a matrix-free approach for matrix-vector products and transfer operators. Additionally, the structural similarities of the lattice cells are exploited through a reduced-order modeling strategy applied locally within each subdomain, enabling efficient local solves within the fine-level smoother. We evaluate the solver's performance through extensive 2D and 3D numerical experiments, demonstrating its efficiency in terms of memory usage, computational time, and robustness under varying mesh refinements, spline degrees, and problem sizes. Notably, we simulate an industrially representative spiral channel regenerative cooling thrust chamber lattice structure — comprising over 66,000 cells — in just minutes using thousands of processes. Finally, we will also present numerical simulations accounting for geometric and material nonlinearities, highlighting the attractiveness of IGA discretizations for such problems.

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Derivatives of tree tensor networks and its applications in Runge–Kutta methods

Jizu Huang

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Tree tensor networks (TTNs) provide a compact and structured representation of high-dimensional data, making them valuable in various areas of computational mathematics and physics. In this talk, we present a rigorous mathematical framework for expressing high-order derivatives of functional TTNs, both with or without constraints. Our framework decomposes the total derivative of a given TTN into a summation of TTNs, each corresponding to the partial derivatives of the original TTN. Using this decomposition, we derive the Taylor expansion of vector-valued functions subject to ordinary differential equation constraints or algebraic constraints imposed by Runge–Kutta (RK) methods. As a concrete application, we employ this framework to construct order conditions for RK methods. Due to the intrinsic tensor properties of partial derivatives and the separable tensor structure in RK methods, the Taylor expansion of numerical solutions can be obtained in a manner analogous to that of exact solutions using tensor operators. This enables the order conditions of RK methods to be established by directly comparing the Taylor expansions of the exact and numerical solutions, eliminating the need for mathematical induction. For a given function $\mathbf{f}$, we derive sharper order conditions that go beyond the classical ones, enabling the identification of situations where a standard RK scheme of order p achieves unexpectedly higher convergence order for the particular function. These results establish new connections between tensor network theory and classical numerical methods, potentially opening new avenues for both analytical exploration and practical computation.

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Novel scalable multilevel solvers for cardiac cell-by-cell model

Ngoc Mai Monica Huynh

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Simulating cardiac cell-by-cell biomechanics involves resolving complex discontinuities at cellular interfaces, such as intercalated disks, and addressing the high computational demands of large-scale simulations. This talk presents a specialized framework employing parallel multilevel solvers, ranging from non-overlapping dual-primal domain decomposition methods, overlapping Schwarz algorithms with Generalized Dryjia-Smith-Widlund (GDSW) preconditioners, to algebraic multigrid methods designed and optimized for composite discontinuous Galerkin (DG) discretizations in cardiac electrophysiology applications at a cellular scale. All the proposed multilevel solvers provide an efficient strategy for accelerating convergence in iterative methods, while effectively managing the discontinuities introduced by heterogeneous cellular structures. Numerical performance evaluations on microscopic cardiac models highlight significant improvements in computational efficiency, with low iteration counts and robust scalability across high-performance computing platforms. Applications include exploring the propagation of electrical waves through discontinuous myocardial tissue, as well as analyzing the effects of structural heterogeneities on conduction dynamics, both on two-dimensional toy cases and on three-dimensional representation of myocardial tissue.

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A highly parallel domain decomposition algorithm for cardiac electro-mechanics simulations

Yi Jiang

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Cardiac electro-mechanics simulations are essential for studying heart function, but they face significant computational challenges due to the strong coupling between electrical activation and tissue mechanics, particularly in patient-specific anatomies. In this talk, we present a highly scalable domain decomposition algorithm for finite element-based cardiac electro-mechanics simulations on unstructured meshes. To enhance robustness during times of strong tissue deformation—where traditional Newton-based solvers may struggle—we introduce nonlinear elimination (NLE) as a nonlinear preconditioning technique. NLE dynamically isolates and resolves localized nonlinearities, stabilizing convergence across the full cardiac cycle. Our method enables efficient strong and weak scaling on patient-specific heart geometries, even at extreme deformations. Benchmark tests demonstrate robust performance, achieving near-ideal parallel scalability on over 16,000 processor cores. The combination of domain decomposition for distributed-memory parallelism and NLE for nonlinear stability makes our approach well-suited for large-scale, high-fidelity cardiac simulations—bringing computational models closer to clinical utility.

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A fast front-tracking approach for a temporal multiscale blood flow problem with a fractional boundary growth

Ping Lin

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In the talk, we consider a typical blood flow problem coupled with slow plaque growth in the artery wall. In the model, the micro (fast) system is the Navier-Stokes equation with a force applied periodically, and the macro (slow) system is a fractional reaction equation, which is used to describe plaque growth with a memory effect. We construct an auxiliary temporal periodic problem and an effective time-average equation to approximate the original problem and analyze the approximation error, where the simple front-tracking technique is used to update the slow moving boundary. An effective multiscale method is then designed based on the approximate problem and the front-tracking framework. We also present a temporal finite difference scheme with a spatial continuous finite element method and analyze its temporal discrete error. Furthermore, a fast iterative procedure is designed to find the initial value of the temporal periodic problem and its convergence is analyzed as well. Our designed front-tracking framework and the iterative procedure for solving the temporal periodic problem make it easy to implement the multiscale method on existing PDE solving software. The temporary multiscale algorithm significantly reduces the computational cost, and the programming effort and its front-tracking framework can easily handle the fluid-structure interaction, especially moving boundaries with more complex geometries.

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Road to "AI for Science": exploring software sustainability through "Couplers"

Kengo Nakajima

The University of Tokyo/RIKEN

"Coupler" is originally a tool for coupling multiple simulation models such as atmosphere and ocean, structure and fluid. In recent years, computer systems and workloads have become more diverse, and the role of couplers in supercomputing has become more important. In this talk, we focus on the "history" of couplers and consider what software sustainability means. We briefly describe three projects. In the 1st project (ppOpen-HPC:2011-2018), we developed an MPI-based scalable coupler for multi-physics simulations. In the 2nd project (h3-Open-BDEC: 2019-2024), we extended the idea of multi-physics coupler for integration of Simulation/Data/Learning (S+D+L) on heterogeneous supercomputer system Wisteria/BDEC-01 by the University of Tokyo, which consists of computing nodes for computational science and engineering with A64FX (Odyssey), and those for Data Analytics/AI with NVIDIA A100 GPU's (Aquarius). The third project (JHPC-quantum: 2023-2028) has started in November 2023, further expanding h3-Open-BDEC to realize Quantum-HPC hybrid computing. In this talk, we will introduce how couplers have evolved and what role they have been playing in supercomputing.

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A low-diffusion positivity-preserving scheme with domain decomposition for high-Mach-number compressible turbulence

Haijun Yu

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Robust and efficient numerical simulation of high-Mach-number compressible turbulence remains a formidable challenge due to intricate shock-turbulence interactions. While significant progress has been made using Euler equations, validated results based on the more physically accurate Navier-Stokes framework remain scarce. Our numerical investigations reveal that the dominant computational difficulty arises from the extreme stiffness of viscous diffusion terms in near-vacuum regions, where conventional explicit time-marching schemes frequently fail. To address this, we propose a novel numerical framework combining two key components: (1) a low-diffusion positivity-preserving discretization for convective fluxes to maintain physical realizability, and (2) a semi-implicit approach coupled with domain decomposition techniques to stabilize the stiff viscous terms. Benchmark tests demonstrate that our methodology effectively suppresses unphysical oscillations caused by vacuum-induced viscosity stiffness, a critical limitation of conventional explicit formulations. Application to the isotropic compressible turbulence simulation with unprecedented turbulent Mach number ($Ma_t > 8$) confirms the scheme's capability to achieve high-fidelity solutions while maintaining numerical stability across multi-scale flow regimes.

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Robust and scalable nonlinear solvers for finite element discretizations of biological transportation networks

Stefano Zampini

KAUST

In this talk, we present robust and scalable, fully implicit nonlinear finite element solvers for the simulations of biological transportation networks driven by the gradient flow minimization of a non-convex energy cost functional. Extensive tests in two and three dimensions demonstrate the robustness and performance of the solver, highlight the sensitivity of the emergent network structures to mesh resolution and topology, and validate the resilience of the linear preconditioner to the ill-conditioning of the model. The implementation achieves near-optimal parallel scaling on large-scale, high-performance computing platforms.

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An efficient Newton-Krylov method with domain decomposition for multi-physics coupling simulations in nuclear reactor power plants

Han Zhang

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The fission reactor encompasses physical processes such as neutron transport in the reactor, fluid flow and heat transfer, multiphase flow, and chemical reactions. It also involves the coupling among devices like the steam generator and the secondary loop of the reactor. It is a typical nonlinear coupled system characterized by multi-physical fields, multi-scales, and multi-components, which has a significant demand for stable, accurate, high-performance, and parallel scalable solution methods. This report focuses on the fully implicit direct simultaneous coupling methods such as Newton-Krylov/ Jacobian-free Newton-Krylov, as well as the special issues in their engineering applications. It also demonstrates the recent progress for the coupled system of the high-temperature gas-cooled reactor power plant based on the parallel fully implicit coupling method.

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MS17 – Efficient solvers for Maxwell equations

Organizers: Victorita Dolean, Alexander Heinlein, Sebastian Kinnewig, Thomas Wick

Maxwell's equations are a system of partial differential equations that describe electromagnetic phenomena, with wide-ranging applications in modern research. Recent developments in physics and engineering, such as in optics, are driving renewed interest in this area. These equations require careful conformal discretizations with appropriate finite elements, presenting challenges for robust and efficient numerical solvers. The resulting algebraic systems can become extremely large due to the oscillatory nature of electromagnetic waves, especially at small wavelengths, where standard solvers for elliptic equations are often insufficient. Consequently, developing efficient iterative solvers for these problems remains a highly active research area.

This mini-symposium will focus on the theoretical and practical aspects of solving Maxwell's equations, with applications to real-world problems and large-scale computations. Newly developed methods and innovative approaches will be discussed to address the challenges posed by these equations.

List of Speakers

- X. Claeys (POems, UMA, ENSTA, Paris) – *Generalized optimized Schwarz method for large scale electromagnetism.*
- V. Dolean (Eindhoven University of Technology) – *Modal analysis of a domain decomposition method for Maxwell's equations in a waveguide.*
- T. Haubold (University of Göttingen) – *Parameter-robust unfitted finite element methods for a Maxwell interface problem.*
- S. Kinnewig (Leibniz University Hannover) – *Coupling deal.II and FROSch: a sustainable and accessible (O)RAS preconditioner.*
- A. Rappaport (ENSTA) – *A hybridizable discontinuous Galerkin method with transmission variables for the time-harmonic Maxwell equations.*
- J. Schoeberl (TU Wien) – *Algebraic multigrid methods for $H(\mathrm{curl})$-elliptic problems.*

Generalized Optimized Schwarz Method for large scale electromagnetism

Xavier Claeys

POems, UMA, ENSTA, Paris

We are interested in the large scale solution to Maxwell's equations in harmonic regime posed in a bounded domain. We consider a reformulation of this problem by means of a Generalized Optimized Schwarz Method, following the strategy described in [1] where Maxwell's equations are solved locally in non-overlapping subdomains, and transmission conditions are imposed across interfaces by means of a so-called exchange operator. In spite of the sign indefiniteness of the initial Maxwell problem, such a formulation enjoys strong coercivity which makes it a favorable setting for classical Krylov solvers such as GMRes. In addition, with an appropriate choice for the exchange operator, the coercivity constant is stable with respect to mesh resolution. The price to pay for this positivity property is that the exchange operator is a priori non-local, which hampers parallelism. In this talk, we shall discuss how this can be circumvented in the specific context of electromagnetism and discuss how our approach can be deployed in a parallel, distributed memory environment.

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Modal analysis of a domain decomposition method for Maxwell's equations in a waveguide

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Time-harmonic wave propagation problems, especially those governed by Maxwell's equations, pose significant computational challenges due to the non-self-adjoint nature of the operators and the large, non-Hermitian linear systems resulting from discretization. Domain decomposition methods, particularly one-level Schwarz methods, offer a promising framework to tackle these challenges, with recent advancements showing the potential for weak scalability under certain conditions. In this work, we analyze the weak scalability of one-level Schwarz methods for Maxwell's equations in strip-wise domain decompositions, focusing on waveguides with general cross sections and different types of transmission conditions such as impedance or perfectly matched layers (PMLs). By combining techniques from the limiting spectrum analysis of Toeplitz matrices and the modal decomposition of Maxwell's solutions, we provide a novel theoretical framework that extends previous work to more complex geometries and transmission conditions. Numerical experiments confirm that the limiting spectrum effectively predicts practical behavior even with a modest number of subdomains. Furthermore, we demonstrate that the one-level Schwarz method can achieve robustness with respect to the wave number under specific domain decomposition parameters, offering new insights into its applicability for large-scale electromagnetic wave problems.

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Parameter-robust unfitted finite element methods for a Maxwell interface problem

Tim Haubold

University of Göttingen

Geometrically unfitted finite element methods such as CutFEM, Finite Cell, XFEM or unfitted DG methods have been developed and applied successfully in the last decades to a variety of problems, ranging from scalar PDEs on stationary domains to systems of PDEs on moving domains and PDEs on level set surfaces. These approaches, combined with established tools of finite element methods, allowed to apply and analyse unfitted methods in many fields. In this talk, we deal with an elliptic interface problem for the time-harmonic quasi-magnetostatic Maxwells equations. Here, the material function μ, the magnetic permeability, can jump at an interface. Such problems are considered in low-frequency applications. Standard unfitted Nitsche methods are not robust with respect to the parameter k, proportional to the wavenumber. For example, a standard Nitsche discretization for the curl-curl-operator introduces terms that do no longer vanish for gradient fields. In this talk, we will use a vectorial finite element discretization based on H(Curl) conforming functions. We will tackle the problem of robustness by using a mixed formulation and a Nitsche formulation. Additionally, we apply a carefully tailored ghost penalization term. A second part of the talk discusses the application of a parameter-robust preconditioner for this interface problem, even with high contrasts.

Joint work with C. Lehrenfeld.

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Coupling deal.II and FROSch: a sustainable and accessible (O)RAS preconditioner

Sebastian Kinnewig

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In this talk, we will present robust and efficient solvers for the Time-Harmonic Maxwell's (THM) equations, with a particular focus on sustainable software development to ensure ease of use. To achieve this, we developed a deal.II-FROSch interface based on the newly introduced Tpetra-based interface to Trilinos, recently added to deal.II. This approach combines the user-friendly design of deal.II with the powerful algebraic preconditioners available through FROSch.

The deal.II-FROSch interface has been extended to include an optimized restricted additive Schwarz (ORAS) preconditioner, which we will discuss in this talk. To validate the ORAS preconditioner, a benchmark problem for the THM equations is proposed, comparing numerical results with solutions derived from Mie's scattering theory. The largest computation, performed on 128 cores, handled over 11 million degrees of freedom, demonstrating the robustness and scalability of the proposed methods.

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A hybridizable discontinuous Galerkin method with transmission variables for the time-harmonic Maxwell equations

Ari Rappaport

ENSTA

Discontinuous Galerkin (DG) finite element methods are well suited for time-harmonic wave propagation problems, thanks to their ability to handle complex geometries and to support arbitrary order approximations with minimal implementation effort. However, at high frequencies the resulting linear systems grow dramatically in size and become ill-conditioned, making direct solvers impractical and standard iterative schemes inefficient.

To overcome these difficulties, hybridizable DG (HDG) methods introduce auxiliary unknowns on element interfaces, enabling the elimination of internal degrees of freedom and yielding a smaller hybrid system. Among these, the CHDG method, initially developed for the Helmholtz equation, defines transmission variables at element interfaces. This leads to a hybrid system that can be interpreted as a non-overlapping Schwarz substructuring domain decomposition method where the subdomains correspond to individual elements.

In this work, we extend the CHDG method to the 3D time-harmonic Maxwell equations. We employ the natural generalization to this context: the transmission variables are vector-valued and live in the tangent space of mesh faces. With this construction we recover the good theoretical properties proven for the Helmholtz equation, namely that the local problems are well posed and the naturally arising fixed point iteration converges unconditionally.

We present details of our implementation in a high-performance parallel `C++` code. Particular care is given to the choice of basis, which we realize implicitly through preconditioning of the linear system. We also compare the performance of different (matrix free) Krylov solvers, as well as the fixed point iteration to solve the hybrid system. We validate our code on a set of benchmarks with known solutions and test the `MPI` parallelism on a large scale test problem from the literature.

Joint work with T. Chaumont-Frelet and A. Modave.

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Algebraic multigrid methods for $H(\mathrm{curl})$-elliptic problems

Joachim Schoeberl

TU Wien

In this talk, we present old and new results for the construction of robust algebraic multigrid methods for magnetostatics and eddy current problems.

The key point of Hiptmair and Arnold-Falk-Winther geometric multigrid methods for Maxwell equations is kernel-preserving smoothers. Here, the de Rham complex is guiding the construction of smoothing blocks. To construct algebraic multigrid preconditioners, one has to preserve exact sequences on the coarser levels. We present the basic principle of this construction, as well as new coarsening criteria for obtaining parameter robustness.

We give a numerical comparison of the performance of various preconditioning methods.

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MS18 – Discrete duality finite volume methods and applications

Organizers: Florence Hubert, Stella Krell, Karol Mikula

This mini-symposium is dedicated to exploring the diverse applications of the Discrete Duality Finite Volume (DDFV) method, originally developed for addressing anisotropic elliptic problems on general meshes. First introduced in the late 1990s, the DDFV method was designed to approximate linear elliptic problems on arbitrary mesh structures. Since its inception, the method has been significantly expanded and adapted to tackle a broad spectrum of complex problems, including nonlinear elliptic problems, convection-diffusion problems, Stokes problems, Navier-Stokes problems, the Peaceman model, Maxwell problems, level-set problems, hyperbolic problems, the Cahn-Hilliard equation, Poisson-Nernst-Planck problems, and Derive-diffusion problems, and more.

The efficiency and versatility of the DDFV method has been rigorously validated through comprehensive benchmarks in two-dimensional (2D) and three-dimensional (3D) contexts, and also for Navier-Stokes (NS) simulations. Beyond its foundational applications, the DDFV technique has been successfully employed across various fields, demonstrating its adaptability and robustness. Notably, it has found applications in biological modeling, such as image smoothing, edge detection, cell migration, and the neurosciences, in the design and optimization of semiconductor devices, in meteorological forecasting and in mathematical finance.

This mini-symposium is an important gathering for the DDFV community, to further explore emerging research and new innovative application areas for the DDFV method. An important focus is also on the parallel solution and Domain Decomposition techniques for the resulting linear systems, which tend to be larger for DDFV discretizations, due to their powerful use of both primal and dual variables. Participants will have the opportunity to share ideas, collaborate on new projects, and contribute to the ongoing evolution of this versatile method.

List of Speakers

- L. Halpern (Université Sorbonne Paris-Nord) – *Convergence analysis of DDFV domain decomposition with Ventcel transmission conditions*.
- G. Lupi (Slovak University of Technology in Bratislava) – *Mathematical and numerical methods for understanding immune cell motion during wound healing*.
- M. Macak (Slovak University of Technology in Bratislava) – *Earth gravity field modelling by using eikonal type boundary condition*.
- F. Nabet (CMAP, Ecole polytechnique) – *Numerical analysis of the discrete duality finite volume method for a Cahn-Hilliard model with surfactants*.

Convergence analysis of DDFV domain decomposition with Ventcel transmission conditions

Laurence Halpern

Université Sorbonne Paris-Nord

This is a follow-up of the plenary talk by Florence Hubert at DD27. We have developed a theory of nonoverlapping domain decomposition for discrete duality finite volumes for anisotropic Laplace equation, using Robin and Ventcel transmission conditions. In rectangular geometry, the DDFV scheme decouples into vertex centered and cell centered schemes, and we applied Fourier analysis to analyse and optimize the convergence for Robin transmission conditions. We also showed that these optimal parameters are also efficient for the more general DDFV scheme.

In this talk, I will present the extension to Ventcel transmission conditions in bounded domains. For this, we will define an adapted DFT, call odd-DFT, which applies to both vertex and cell centered scheme, but also to the coupled scheme when the matrix of anisotropy is not diagonal, resulting into a coupled cell/vertex centered scheme.

This is a joint work with M. Gander, F. Hubert, and S. Krell. See also Minisymposium 9 the talk by Stella Krell on overlapping DD.

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Mathematical and numerical methods for understanding immune cell motion during wound healing

Giulia Lupi

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We propose a new workflow to analyze macrophage motion during wound healing. These immune cells play a critical role in tissue repair, as they are attracted to the wound site after an injury. Their motion is a combination of directional movement and random motion. Therefore, we begin by smoothing the original trajectories. The smoothing model is based on curve evolution approach, where the curve evolves under the influence of two key terms: a smoothing term, determined by the local curvature of the trajectory, and an attracting term, which ensures that the curve stays close to the original trajectory. This model allows us to calculate the velocities on the smoothed trajectories; we then use them as sparse samples to reconstruct the wound attractant field. This process involves solving a minimization problem for the vector components and lengths of the velocity field. The solution reduces to solving the Laplace equation with Dirichlet boundary conditions on the sparse samples and zero Neumann boundary conditions on the domain boundary. The result is a vector field where the direction and lengths of the vectors are interpolated/extrapolated from the Dirichlet conditions. We present the numerical methods used to discretize the boundary value problem obtained from solving the minimization problem to reconstruct the velocity vector field and the boundary value problems considered for post-processing the reconstructed vector field.

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Earth gravity field modelling by using eikonal type boundary condition

Marek Macak

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In this contribution, we present a novel concept for the determination of the Earth's gravity field by solving the nonlinear satellite-fixed geodetic boundary value problem (NSFGBVP). The NSFGBVP consists of the Laplace equation defined in the 3D bounded domain outside the Earth, the nonlinear eikonal-type boundary condition (BC) prescribed on the Earth's surface, and the Dirichlet BC given on a spherical boundary placed approximately at the altitude of satellite mission and additional four side boundaries. Since this formulation of the problem includes also a generalized eikonal BC for the norm of the gradient of the gravity potential on the Earth's surface, solution to this problem provides possibilities for application in other similarly defined problems. Our proposed solution approach is based on an iterative approach in which in every iteration we solve numerically an oblique derivative boundary value problem for a disturbing potential by using the DD method. Finally, we discuss the obtained numerical results.

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Numerical analysis of the discrete duality finite volume method for a Cahn-Hilliard model with surfactants

Flore Nabet

CMAP, Ecole polytechnique

This work focuses on the numerical analysis of a phase field model with surfactants (or surface active agents). Surfactants are amphiphilic molecules characterized by a hydrophilic head group and a hydrophobic tail. The presence of surfactants may greatly affect the physical properties of fluid mixtures. Indeed, these effects are used critically in many important applications in everyday lives. The model is based on two Cahn–Hilliard equations: the first to describe the phase separation dynamics between water and air, and the second to take account of the dynamic of surfactants. The coupling potential between these equations is constructed in such a way as to take into account the physical properties of the model. In addition to the difficulties due to the Cahn-Hilliard model, there are very strong nonlinearities due to the coupling potential. Our main objective is to prove the convergence of the discrete solution to the continuous one using a DDFV scheme that offers a great robustness.

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MS19 – Domain decomposition methods and model order reduction techniques

Organizers: Simone Deparis, Marco Discacciati, Paola Gervasio, Matteo Giacomini

Numerical simulations of real-life engineering systems for predictive purposes and optimal control typically require multiple queries to complex and computationally demanding solvers. Reduced order methods can be employed to reduce the dimensionality of parametric problems, but the cost of constructing physics-based surrogate models still poses a challenge in the presence of large-scale, multi-physics systems. Efficiently combining model order reduction with domain decomposition methods can help to overcome this computational barrier. This minisymposium will provide a forum to present recent developments on advanced techniques to arbitrarily couple reduced order and surrogate models, full order models, and data-driven methods, particularly for applications to multi-physics systems and multi-fidelity simulations.

List of Speakers

- S. Deparis (EPFL) – *A self-adaptive IMEX time integration scheme with applications to the Navier-Stokes equations and domain decomposition*.
- M. Giacomini (Universitat Politècnica de Catalunya, Centre Internacional de Mètodes Numèrics en Enginyeria - CIMNE) – *Domain decomposition and data augmentation to engineer parsimonious surrogate models*.
- T. Hagstrom (Southern Methodist University) – *Domain decomposition and the reduced order modeling of wave-dominated systems*.
- P. Kuberry (Sandia National Laboratories) – *Minimally intrusive data-driven approximation of Schur complement-based coupling operators for heterogeneous numerical methods*.
- R. Rubio (CIMNE/UPC) – *Preconditioning iterative solvers via empirical finite element method (EIFEM)*.
- T. Taddei (Inria) – *Optimization-based model order reduction of fluid structure interaction problems*.
- M. Torzoni (Politecnico di Milano) – *Physics-data structural optimization: from latent spaces to member composition*.

A self-adaptive IMEX time integration scheme with applications to the Navier-Stokes equations and domain decomposition

Simone Deparis

EPFL

We present a novel automatic implicit-explicit (IMEX) time integration scheme tailored to algebraic differential equations arising from spatial discretizations by finite difference methods. The proposed approach exploits the reduced basis method to dynamically partition the system into stiff and non-stiff components, enabling efficient and stable time integration without requiring manual tuning of the splitting.

We rigorously analyze the stability and convergence of the method, and demonstrate its accuracy through numerical experiments on representative model problems.

Building on this foundation, we extend the scheme to accommodate finite element discretizations and domain decomposition techniques. This generalization makes the method particularly well suited for large-scale simulations of complex systems. In particular, we show its applicability to the time integration of the incompressible Navier–Stokes equations, illustrating its potential for use in fluid dynamics problems.

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Domain decomposition and data augmentation to engineer parsimonious surrogate models

Matteo Giacomini

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Surrogate models of parametric systems require the solution of multiple instances of a physical problem, either to populate a dataset with representative snapshots for a posteriori reduced order models (ROMs) or to sequentially construct a reduced basis in a priori ROMs. These operations are computationally intensive and can be unaffordable when large-scale domains and nonlinear models are involved. This talk will discuss some recent solutions to tackle such computational challenges. First, an overlapping Schwarz method will be presented for parametric linear problems. This approach introduces a few additional parameters to describe the trace of the solution at the subdomain interfaces and exploits the a priori framework of proper generalised decomposition to construct physics-based local surrogate models. The resulting ROM avoids introducing Lagrange multipliers or solving extra problems in the online phase to couple the subdomains [1,2]. In the context of nonlinear parametric problems, data augmentation strategies will be presented to engineer artificial snapshots from a few computed full-order solutions, in order to enrich the dataset for the proper orthogonal decomposition projection ROM. This is achieved by appropriately combining existing snapshots while guaranteeing mass and momentum conservation principles [3]. In both circumstances, compliance with physical laws is enforced in the reduced space by construction. Numerical examples, including parametric thermal problems and viscous laminar incompressible flows, will be presented to illustrate the computational advantages of the discussed domain decomposition and data augmentation strategies for the construction of parsimonious surrogate models.

Joint work with P. Díez, M. Discacciati, E. Evans, A. Muixí, and S. Zlotnik.

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Domain decomposition and the reduced order modeling of wave-dominated systems

Thomas Hagstrom

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Large scale systems with complex dynamical behavior are often beyond existing computational capabilities, particularly in cases where multiple simulations are needed to solve inverse or design problems. Reduced order modeling can enable the practical computational treatment of such systems. However, for multicomponent wave-dominated systems accurate global reduced order models typically cannot be constructed. Here we discuss domain decomposition as a way to circumvent this issue. In particular, we consider methods which focus on modeling the fluxes between subdomains which are large relative to the wavelengths. In principle, such models can compress the dimensionality of bases needed to accurately approximate the fields within each subdomain. Specific examples, including linear and weakly nonlinear wave propagation in highly heterogeneous media, are presented. Our reduced order models are constructed based on the resolvent of the linearized operator in each subdomain and, depending on the particular problem, can be built in an embarrassingly parallel way. We then leverage standard reduced order modeling techniques from the control theory literature, based in general on the rational approximation in frequency space of the interface operators, which map incoming characteristic data to outgoing characteristic data. For nonlinear problems, we consider simple methods incorporating polynomial nonlinearities in an operator inference framework.

Joint work with M. DeVernon.

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Minimally intrusive data-driven approximation of Schur complement-based coupling operators for heterogeneous numerical methods

Paul Kuberry

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Lagrange multiplier-based (LM) coupling approaches have been shown to be stable, accurate, and applicable to multiphysics problems coupled over an interface. For explicitly time-integrated coupled problems (IVR), the resulting Schur complement that is used to solve for the coupling traction forces is tractable to form but requires access to operators generally embedded deeply within the software modeling the subdomain. For implicitly time-integrated problems (IFR), the Schur complement involves inversion of stiffness matrices and is not computationally feasible. Through a data-driven discovery of coupling operators, we are able to loosen the requirement for access to multiple complex data structures. The resulting operator is more compact and less computationally expensive to evaluate than the full-order coupling operator.

With the rise in popularity of data-driven modeling techniques applied to subdomain problems, we make the assumption of the availability of full-resolution subdomain solution data and leverage an idea by Carey et. al. [1] from which to generate high-order approximations of traction forces for snapshots. We use the traction force snapshots to generate a reduced basis and then perform standard least-squares operator regression to approximate the Schur complement system. The approach is minimally burdensome, requiring only Gramian matrices capturing a surface integral of solution trial and LM test functions over the interface.

We demonstrate the effectiveness of the technique with several numerical experiments and make comparison, with respect to speed, against Schwarz-based approaches and a full-order Schur complement based approach.

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Preconditioning iterative solvers via empirical finite element method (EIFEM)

Raul Rubio

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Recent advancements in additive manufacturing and topology optimization enabled the design of complex lattice structures with applications across various industries, including aerospace, automotive, or chemical engineering. However, the computational cost of solving these structures using the Finite Element Method (FEM) can be prohibitive, particularly when dealing with large-scale systems arising from 3D geometries. The high memory and time requirements of direct solvers make them unsuitable for such problems, leaving efficient iterative solvers as the only viable alternative.

To mitigate these computational challenges, Reduced Order Models (ROMs) have emerged as a promising solution, providing faster simulations, though at the expense of losing accuracy. In this study, we propose a novel approach by integrating the multiscale ROM Empirical Interscale Finite Element Method (EIFEM) as a preconditioner within the conjugate gradient iteration. The strategy can be viewed as an alternative to the two-grid V-cycle for cellular structures, where the coarse mesh is replaced by EIFEM. We present a series of numerical experiments in both the linear and nonlinear regimes to validate the performance of this approach, demonstrating its potential to enhance the efficiency of numerical simulations in real-world applications.

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Optimization-based model order reduction of fluid structure interaction problems

Tommaso Taddei

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We introduce optimization-based full-order and reduced-order formulations of fluid structure interaction problems. We consider the flow of an incompressible Newtonian fluid which interacts with an elastic body: we consider an arbitrary Lagrangian Eulerian formulation of the fluid problem and a fully Lagrangian formulation of the solid problem; we rely on a finite element discretization of both fluid and solid equations.

The distinctive feature of our approach is an implicit coupling of fluid and structural problems that relies on the solution to a constrained optimization problem with equality constraints. We discuss the application of projection-based model reduction to both fluid and solid subproblems. We further discuss the issue of energy stability and the application of space-time formulations to enhance stability and reduce online costs.

Joint work with X. Xu and L. Zhang.

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Physics-data structural optimization: from latent spaces to member composition

Matteo Torzoni

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Designing high-performing structural systems requires balancing strict mechanical requirements with practical constraints related to fabrication, transportation, and construction. This talk presents two machine learning-based frameworks for scalable, data-driven structural optimization. While grounded in learned representations and structured decision-making, both approaches echo classical computational mechanics techniques – namely, model order reduction and domain decomposition. The first method targets real-time topology optimization through a two-stage surrogate modeling pipeline [1]. Inspired by reduced-order modeling of parametrized differential problems, it projects high-dimensional optimal topologies onto a low-dimensional latent manifold using a deep autoencoder. A neural surrogate maps design parameters to this latent space, enabling the decoder to reconstruct both topology and stress fields in real time, without the need for iterative solvers. Here, optimization is performed offline using a dataset of precomputed solutions, trading costly training for virtually instantaneous inference. The second framework addresses truss optimization via a generative formulation based on grammar-constrained Markov decision processes [2]. In this setting, optimization is intertwined with learning: the design policy improves while sequentially assembling truss members under grammar rules that encode engineering feasibility. These constraints decompose the design space into meaningful subregions, enabling efficient exploration of design trajectories. This strategy mirrors domain decomposition in spirit, as the final structure is assembled from interpretable base components, analogous to subdomain solutions. These methods demonstrate how integrating engineering principles with modern machine learning yields efficient, high-quality structural designs. We showcase their computational advantages and robustness through several examples, particularly in progressive construction scenarios.

Joint work with A. Corigliano and L. Rosafalco

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MS20 – Localized model order reduction, multiscale and domain decomposition methods

Organizers: Martin J. Gander, Mario Ohlberger, Stephan Rave

Projection-based parametric model order reduction (MOR) accelerates the solution of parameterized PDEs by performing an additional (Petrov-)Galerkin projection step of the discretized system equations onto a problem-adapted reduced space. This reduced space is computed from solutions of the full-order system for certain well-selected snapshot parameters.

For large-scale systems with localized parameter influence, however, computing a suitable global reduced space is infeasible. Localized MOR techniques address this challenge by computing spatially localized reduced spaces from localized full-order solutions. These localized spaces are then glued together to form a global approximation space.

This approach shares many similarities with the construction of spectral coarse spaces in domain decomposition methods. At the same time, similar ideas are prevalent in numerical multiscale methods like the (generalized) multiscale finite-element method (MsFEM) or the localized orthogonal decomposition (LOD). It is the goal of this minisymposium to bring together researchers from all three fields, in order to exchange recent ideas and foster further collaboration.

List of Speakers

- K. Brenner (Université Côte d'Azur) – *Multiscale and domain decomposition methods for linear and nonlinear fracture network flows.*
- D. Kolombage (University of Bonn) – *Offline–online decomposition for randomly perturbed coefficients.*
- R. Maier (Karlsruhe Institute of Technology) – *A higher-order localized orthogonal decomposition strategy.*
- G. Li (The University of Hong Kong) – *On edge multiscale space based hybrid Schwarz preconditioner for Helmholtz problems with large wavenumbers.*
- S. Rave (University of Münster) – *Parareal with spectral coarse solvers.*
- M. Schlottbom (University of Twente) – *An extension of the approximate component mode synthesis method to the heterogeneous Helmholtz equation.*

Multiscale and domain decomposition methods for linear and nonlinear fracture network flows

Konstantin Brenner

Université Côte d'Azur

Faults and fractures play a major role in subsurface hydraulics by providing conductivity for mass and energy transfer through otherwise low-permeability geological formations. Depending on the application, this additional conductivity is viewed either as a threat, as in the case of nuclear waste or CO2 storage applications, or as an opportunity, as in applications in deep geothermal energy extraction. Either way, understanding flow patterns in the networks of fractures is of fundamental importance.

In this talk, we will consider single-phase flow in fracture networks represented as collections of intersecting planar polygons leading to so-called Discrete Fracture Network models. The flow model is derived either from the linear Darcy law or the nonlinear Darcy-Forchhimer law. To efficiently handle large fracture networks, we propose a numerical multiscale method, in which macro-cells are associated with either individual fractures or clusters of fractures. Our approach is based on approximate substructuring, where the interior of the macro-cells is eliminated based on a low-dimensional parametrization of Dirichlet data.

Through numerical experiments with networks containing tens of thousands of individual fractures, we demonstrate the ability of our multiscale approach to accurately capture large-scale flow patterns at a marginal computational cost compared to fine-scale calculations. Furthermore, combining this multiscale method with overlapping domain decomposition yields an efficient iterative method.

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Offline–online decomposition for randomly perturbed coefficients

Dilini Kolombage

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Multiscale and random perturbations frequently arise in the modelling of fine-scale imperfections in modern composite and structured materials. The Localized Orthogonal Decomposition (LOD) method is a multiscale technique that enables localized treatment of heterogeneities. However, when tackling elliptic eigenvalue problems across multiple realizations of randomly perturbed coefficients, a direct application of LOD becomes computationally intensive due to the repeated construction of localized correctors.

To address this, we explore an offline-online decomposition framework inspired by [2], in which local contributions to the LOD stiffness matrix are precomputed based on a collection of reference configurations. This allows the precomputed local contributions-restricted to subdomains or patches-to be reused across realizations, eliminating the need to reconstruct multiscale spaces globally for each perturbation.

Building on this idea, we propose in [1] a Petrov-Galerkin variant of the LOD method, adapted to the offline-online paradigm, designed for elliptic eigenvalue problem with periodic boundary conditions and random multiscale defects. We further introduce a dynamic online correction mechanism that adapts to stochastic variability, enhancing approximation accuracy particularly at moderate to higher defect probabilities. We present a priori error estimates for both eigenvalues and eigenfunctions, and support our analysis with numerical experiments demonstrating both the convergence behaviour and the computational efficiency of our approach.

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A higher-order localized orthogonal decomposition strategy

Roland Maier

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This talk is about the construction of higher-order multiscale methods in the framework of the localized orthogonal decomposition approach. We show how to achieve higher-order convergence rates in the elliptic setting without restrictive regularity assumptions on the domain, the coefficient, or the exact solution. Further, we discuss extensions to time-dependent problems where appropriate adaptations are required. Numerical examples are presented to illustrate the theoretical findings.

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On edge multiscale space based hybrid Schwarz preconditioner for Helmholtz problems with large wavenumbers

Guanglian Li

The University of Hong Kong

We develop a novel hybrid Schwarz method, termed as edge multiscale space based hybrid Schwarz (EMs-HS), for solving the Helmholtz problem with large wavenumbers. The problem is discretized using H^1-conforming nodal finite element methods on meshes of size h decreasing faster than k^{-1} such that the discretization error remains bounded as the wavenumber increases. EMs-HS consists of a one-level Schwarz preconditioner (RAS-imp) and a coarse solver in a multiplicative way. The RAS-imp preconditioner solves local problems on overlapping subdomains with impedance boundary conditions in parallel, and combines the local solutions using partition of unity. The coarse space is an edge multiscale space proposed in [1]. The key idea is to first establish a local splitting of the solution over each subdomain by a local bubble part and local Helmholtz harmonic extension part, and then to derive a global splitting by means of the partition of unity. This facilitates representing the solution as the sum of a global bubble part and a global Helmholtz harmonic extension part. We prove that the EMs-HS preconditioner leads to a convergent fixed-point iteration uniformly for large wavenumbers, by rigorously analyzing the approximation properties of the coarse space to the global Helmholtz harmonic extension part and to the solution of the adjoint problem. Distinctly, the theoretical convergence analysis is valid in two extreme cases: using minimal overlapping size among subdomains (of order h), or using coarse spaces of optimal dimension (of magnitude k^d, where d is the spatial dimension). We provide extensive numerical results on the sharpness of the theoretical findings and also demonstrate the method on challenging heterogeneous models.

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Parareal with spectral coarse solvers

Stephan Rave

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We present a new class of Parareal algorithms for parabolic PDEs, where the coarse solver for each interval in the time domain decomposition is given by a spectral approximation of the transfer operator mapping initial values at the beginning of the interval to the solution at its end. As high-frequency oscillations are quickly damped over time, these transfer operators exhibit a fast singular value decay, so a low-rank approximation is possible. By leveraging randomized singular value decomposition, such low-rank approximations are obtained embarrassingly parallel by computing local fine solutions for random initial values. We provide a posteriori error bounds for the Parareal approximation error in terms of norms of local updates and the computed singular values of the transfer operators. Our numerical experiments show that our approach can significantly outperform Parareal with basic single-step coarse solvers. At the same time, it allows to further increase parallelism in Parareal by trading coarse updates by a larger number of local solutions.

Joint work with M.J. Gander and M. Ohlberger.

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An extension of the approximate component mode synthesis method to the heterogeneous Helmholtz equation

Matthias Schlottbom

University of Twente

We discuss an extension of the approximate component mode synthesis (ACMS) method to the two-dimensional heterogeneous Helmholtz equation. The ACMS method has originally been introduced by Hetmaniuk and Lehoucq as a multiscale method to solve elliptic partial differential equations. The ACMS method uses a domain decomposition to separate the numerical approximation by splitting the variational problem into two independent parts: local Helmholtz problems and a global interface problem. While the former are naturally local and decoupled such that they can be solved in parallel, the latter requires the construction of suitable local basis functions. Here, we consider suitable extensions of local eigenmodes to approximate the interface problem. We present an error analysis of this approach focusing on the case where the domain decomposition is kept fixed, but the number of eigenfunctions is increased. The theoretical results are supported by numerical experiments verifying algebraic convergence for the method.

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MS21 – Multilevel domain decomposition methods and coarse spaces

Organizers: Martin J. Gander, Alexander Heinlein

Coarse spaces have undergone intense development over the last decade, leading to tremendous progress compared to the initial coarse spaces, such as the piecewise constant coarse space of Nicolaides, which provided scalability but little more. Modern coarse spaces not only ensure scalability but are also tightly integrated with the underlying solver to better compensate for the solver's challenges in addressing specific physical problems. These challenges include heterogeneities, anisotropy, incompressibility, and more. As problem sizes increase, the coarse problem in two-level domain decomposition methods is increasingly becoming a bottleneck, necessitating the use of more than two levels—particularly in parallel computations, where the coarse solve typically limits parallel scalability. In addition to traditional approaches, new and exciting directions include the algebraic construction of coarse spaces and the use of machine learning algorithms to learn effective coarse spaces. This minisymposium aims to bring together researchers working in this exciting field, providing them with a platform to present their latest findings and engage in discussions with one another.

List of Speakers

- P. Bastian (Heidelberg University) – *Robust coarse spaces for overlapping domain decomposition methods*.
- M. Doškář (Czech Technical University in Prague) – *Reduced basis for accelerating adaptive coarse space construction in FETI-DP*.
- M. Fry (University of Strathclyde) – *Using spectral coarse spaces of the H-GenEO type for efficient solutions of the Helmholtz equation*.
- J. Galvis (Universidad Nacional de Colombia) – *On overlapping domain decomposition methods for high-contrast multiscale problems*.
- F. Nataf (Laboratoire J.L. Lions) – *Mixed precision analysis of the of the additive Schwarz method preconditioner with a GenEO coarse space*.
- L. Theisen (RWTH Aachen University) – *A scalable two-level domain decomposition eigensolver for periodic Schrödinger eigenstates in anisotropically expanding domains*.
- S. Van Criekingen (CNRS (IDRIS and Maison de la Simulation)) – *Advances in one- and two-level parallel substructured Schwarz methods*.
- F. Wang (Xi'an Jiaotong University) – *Overlapping Schwarz preconditioners for randomized neural networks with domain decomposition*.

Robust coarse spaces for overlapping domain decomposition methods

Peter Bastian

Heidelberg University

Recent years have seen a tremendous development of robust coarse spaces for domain decomposition (DD) methods which rely on solving local eigenproblems. In this contribution we focus on two of these methods for overlapping DD, which are the "generalized eigenproblems in the overlaps" (GENEO) and "multiscale generalized finite elements" (MSGFEM) used as preconditioner. After introducing these methods within a joint framework we discuss the extension to more than two levels (for GENEO) as well as the reduction of the complexity of the eigenproblem by only solving it in the ring-shaped overlap region. This new method is supported by theoretical as well as numerical results.

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Reduced basis for accelerating adaptive coarse space construction in FETI-DP

Martin Doškář

Czech Technical University in Prague

Among the family of non-overlapping domain decomposition methods, the Dual-Primal Finite Element Tearing and Interconnecting method (FETI-DP) of Farhat et al. [1] has gained prominence for its versatility, robustness, and scalability. However, in problems with high contrast in material properties, and particularly when discontinuities align with subdomain interfaces, its performance can degrade significantly. While advanced scaling techniques, such as rho- and Deluxe-scaling, mitigate these issues to some extent, they are not always sufficient. To recover robustness of the iterative scheme, the coarse space must often be enriched beyond the standard set of primal degrees of freedom (typically chosen subdomain corners). Early enrichments such as interface averages, higher-order moments, or coefficient-weighted averages improved performance, but lacked universality. A more systematic strategy came with the adaptive coarse spaces pioneered by Mandel and Sousedík [2], which identify problematic interface modes via local generalized eigenvalue problems (GEVPs). While offering provable robustness, solving a GEVP for each interface is computationally intensive, and in many cases, the cost outweighs the benefit. Nonetheless, these adaptive approaches offer critical insight into what constitutes an effective coarse space. Building on this and heuristics such as Frugal FETI-DP [3], we propose a strategy that exploits coefficient distributions along interfaces to construct a reduced basis for the GEVPs used in adaptive coarse space construction. Through numerical examples—from academic binary distributions to problems in modular topology optimization—we show that RB-GEVP achieves near-equivalent mode detection using GEVPs that are orders of magnitude smaller. The resulting condition numbers remain comparable to those obtained by full adaptivity, but at a fraction of the computational cost [4].

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Using spectral coarse spaces of the H-GenEO type for efficient solutions of the Helmholtz equation

Mark Fry

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The Helmholtz equation is a widely used model in wave propagation and scattering problems. However, its numerical solution can be computationally expensive in high-frequency regime due to the oscillatory solution and the potential contrasts in coefficients. Parallel domain decomposition methods have been identified as promising solvers for such problems, but they often require a suitable coarse space to achieve robust behaviour. In this talk, we present the H-GenEO coarse space, which constructs an effective coarse space using localized eigenvectors of the Helmholtz operator. While the GenEO coarse space is designed for symmetric positive definite problems, the theory cannot be extended directly to the H-GenEO coarse space due to the indefinite nature of the underlying problem. During this talk it will be shown what the H-GenEO coarse space is capable of providing the required robust behaviour when used with a suitable domain decomposition method. Numerical experiments for increasing wave numbers demonstrate the efficiency of the method in solving complex Helmholtz problems, with potential applications in various scientific and engineering domains.

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On overlapping domain decomposition methods for high-contrast multiscale problems

Juan Galvis

Universidad Nacional de Colombia

In this talk, we review the domain decomposition methods that use generalized multiscale finite element methods targeting problems with high-contrast multiscale coefficients. We review some successful cases and also the case of many high contrast channels, that is still a challenging case.

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Mixed precision analysis of the of the additive Schwarz method preconditioner with a GenEO coarse space

Frédéric Nataf

Laboratoire J.L. Lions, Paris, France

This work is concerned with domain decomposition preconditioners, specifically the Schwarz method with GENEO coarse space, to solve large, sparse symmetric positive definite problems. Through libraries such as HPDDM, these preconditioners have already been efficiently parallelized in their numerical implementations. However, they still require expensive linear algebra operations in each local subdomain. Motivated by the emergence of fast low precision arithmetic in hardware, we aim in this work to speed them up using mixed precision. To do this, we need to identify the sensitivity of these operations to perturbation and propose an actionable criteria for selecting the appropriate precision for each local subdomain. In order to do so, we develop a perturbation theory for our preconditioner to bound the worst-case loss of efficiency of the preconditioner and study the sharpness of this theoretical bound through numerical experiments with the FreeFEM, petsc4py, and HPDDM libraries. Our findings show that the only important parameter is the maximum of the sizes of the local subdomain perturbations, weighted by the condition number of the local subdomain matrix. Our results therefore suggest that preconditioners can be constructed in mixed precision while effectively controlling the loss of efficiency.

Joint work with T. Caruso, P. Jolivet, T. Mary, and P.-H. Tournier

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A scalable two-level domain decomposition eigensolver for periodic Schrödinger eigenstates in anisotropically expanding domains

Lambert Theisen

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Accelerating iterative eigenvalue algorithms is often achieved by employing a spectral shifting strategy. Unfortunately, improved shifting typically leads to a smaller eigenvalue for the resulting shifted operator, which in turn results in a high condition number of the underlying solution matrix, posing a major challenge for iterative linear solvers. This paper introduces a two-level domain decomposition preconditioner that addresses this issue for the linear Schrödinger eigenvalue problem, even in the presence of a vanishing eigenvalue gap in non-uniform, expanding domains. Since the quasi-optimal shift, which is already available as the solution to a spectral cell problem, is required for the eigenvalue solver, it is logical to also use its associated eigenfunction as a generator to construct a coarse space. We analyze the resulting two-level additive Schwarz preconditioner and obtain a condition number bound that is independent of the domain's anisotropy, despite the need for only one basis function per subdomain for the coarse solver. Several numerical examples are presented to illustrate its flexibility and efficiency.

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Advances in one- and two-level parallel substructured Schwarz methods

Serge Van Criekingen

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Substructured Schwarz methods are interpretations of classical volume Schwarz methods as algorithms on interface variables. We introduce here a parallel algebraic trace characterization to supersede the geometric identification of the substructure within our PETSc implementation of the Parallel Schwarz Method (PSM, equivalent to RAS). We moreover discuss a computationally efficient way to implement a two-level substructured method with coarse space functions defined on the skeleton, and present numerical results to compare substructured and classical volume methods.

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Overlapping Schwarz preconditioners for randomized neural networks with domain decomposition

Fei Wang

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Randomized neural networks (RaNNs), characterized by fixed hidden layers after random initialization, offer a computationally efficient alternative to fully parameterized neural networks trained using stochastic gradient descent-type algorithms. In this talk, we integrate RaNNs with overlapping Schwarz domain decomposition in two primary ways: firstly, to formulate the least-squares problem with localized basis functions, and secondly, to construct effective overlapping Schwarz preconditioners for solving the resulting linear systems. Specifically, neural networks are randomly initialized in each subdomain following a uniform distribution, and these localized solutions are combined through a partition of unity, providing a global approximation to the solution of the partial differential equation. Boundary conditions are imposed via a constraining operator, eliminating the necessity for penalty methods. Furthermore, we apply principal component analysis (PCA) within each subdomain to reduce the number of basis functions, thereby significantly improving the conditioning of the resulting linear system. By constructing additive Schwarz (AS) and restricted AS preconditioners, we efficiently solve the least-squares problems using iterative solvers such as the Conjugate Gradient (CG) and generalized minimal residual methods. Numerical experiments clearly demonstrate that the proposed methodology substantially reduces computational time, particularly for multi-scale and time-dependent PDE problems. Additionally, we present a three-dimensional numerical example illustrating the superior efficiency of employing the CG method combined with an AS preconditioner over direct methods like QR decomposition for solving the associated least-squares system.

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MS22 – Robust and efficient domain decomposition methods for multiscale and multi-physics problems

Organizers: Eric Chung, Hyea Hyun Kim

Many scientific and engineering applications involve mathematical models whose solutions have multiple scales in space and time. Solving these problems require carefully designed coarse spaces that can be used with some domain decomposition methods. The choice of the coarse spaces is very challenging for more complex multi-physics problems. In this mini-symposium, the speakers will present some recent advances in this field and related areas.

List of Speakers

- T. Bevilacqua (Universität zu Köln) – *Highly scalable monolithic Schwarz preconditioners for thermo-elastoplastic laser beam welding simulations*.
- J. G. Calvo (Universidad de Costa Rica) – *Additive Schwarz preconditioning for discontinuous virtual element discretizations with irregular subdomains and high-contrast coefficient*.
- V. Dolean (Eindhoven University of Technology) – *Domain decomposition preconditioners and multi-scale approaches to solve stationary and time-dependent nonlinear equations*.
- A. Heinlein (Delft University of Technology) – *Advances of FROSch preconditioners for multiphysics and multiscale simulations*.
- L. Holbach (Heidelberg University) – *A robust two-level restricted additive Schwarz preconditioner based on multiscale spectral generalized FEM for high-Péclet number heterogeneous convection–diffusion*.
- J. Huang (Xiangtan University) – *Multicontinuum homogenization for coupled flow and transport equations*.
- W. T. Leung (City University of Hong Kong) – *High-permeability focusing coarse spaces for overlapping Schwarz preconditioners in high-contrast time-dependent PDEs*.
- O. Widlund (Courant Institute, New York) – *An adaptive BDDC method using vertex-based generalized eigenvalue problems*.

Highly scalable monolithic Schwarz preconditioners for thermo-elastoplastic laser beam welding simulations

Tommaso Bevilacqua

Universität zu Köln

In this talk, we present a robust and scalable parallel solver for nonlinear and time-dependent thermo-elastoplastic simulations arising in laser beam welding (LBW) processes. The LBW problem is modeled as a multiphysics system involving tightly coupled temperature and displacement fields, governed by a system of nonlinear partial differential equations. These equations are discretized in space using finite elements and in time via the backward Euler method, with Newton's method employed for linearization. The resulting sequence of large-scale, ill-conditioned linear saddle-point problems poses significant challenges for iterative solvers.

To accelerate the convergence of Krylov subspace methods such as GMRES, we employ a monolithic, two-level overlapping Schwarz domain decomposition preconditioner. This class of preconditioners applies the decomposition directly to the full coupled system, preserving the multiphysics structure throughout both local and coarse levels. We explore and implement several variants of the Generalized Dryja–Smith–Widlund (GDSW) coarse space—namely GDSW, RGDSW, and GDSW*—which are designed to enhance robustness and scalability across a large number of subdomains. Particular attention is given to strategies for reducing coarse space dimensionality, which becomes critical in 3D simulations with many subdomains.

The developed methods are implemented efficiently within the PETSc framework, ensuring portability and high performance on modern parallel architectures. We incorporate advanced strategies such as adaptive time stepping, coarse matrix truncation, and solver component recycling across Newton iterations to further enhance performance. We evaluate and compare different combinations of coarse spaces across displacement and temperature fields, analyzing their effectiveness in maintaining solver robustness.

Extensive numerical experiments on realistic LBW problems demonstrate excellent scalability and efficiency, confirming the suitability of our approach for large-scale, high-fidelity thermo-mechanical simulations. Overall, this work helps to make domain decomposition methods more practical and efficient for solving large, complex multiphysics problems.

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Additive Schwarz preconditioning for discontinuous virtual element discretizations with irregular subdomains and high-contrast coefficient

Juan G. Calvo

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We present a two-level additive Schwarz approach for preconditioning linear systems arising from discontinuous Galerkin formulations of the Virtual Element Method. This approach accommodates irregular subdomains, offering greater flexibility compared to previous techniques. We consider the two-dimensional Poisson equation as a model problem and provide numerical results that confirm the robustness of the preconditioner in the presence of large jumps in the diffusion coefficient. The approach demonstrates good scalability, with the condition number remaining consistent with theoretical bounds established in previous studies for both conforming and nonconforming discretizations, particularly when the coefficient jumps align with subdomain interfaces. Furthermore, we show that enriching the coarse space with functions derived from local eigenvalue problems improves performance for the case of high-contrast multiscale coefficients.

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Domain decomposition preconditioners and multi-scale approaches to solve stationary and time-dependent nonlinear equations

Victorita Dolean

Eindhoven University of Technology

In this contribution, we build on a previous work where we introduced a coarse space for the Poisson equation posed on the perforated domains containing multiscale features, as they arise in simplified flow models in an urban environment. Here, the focus is on left nonlinear preconditioning techniques based on overlapping subdomains, implementing techniques that use the coarse space proposed in the linear case to provide scalability. The coarse space was used in combination with the RAS preconditioner, an overlapping domain decomposition technique for the solution of linear problems. Here, we compare numerically different preconditioning strategies for a given model problem. While the coarse space was originally based on the linear Poisson equation, we find that it is a fitting coarse space for nonlinear problems as well.

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Advances of FROSch preconditioners for multiphysics and multiscale simulations

Alexander Heinlein

Delft University of Technology

FROSch (Fast and Robust Overlapping Schwarz), part of the Trilinos software framework, provides scalable multilevel Schwarz preconditioners aimed at algebraic construction for solving large-scale systems of partial differential equations. This talk reports on recent developments in FROSch, with a focus on improving robustness and efficiency for block-structured systems and strongly heterogeneous problems.

Several strategies have been implemented to address these challenges, including the construction of monolithic and spectral coarse spaces that capture the structure of the underlying physical models. In addition, enhancements to the communication patterns between consecutive levels of the multilevel hierarchy have been implemented, reducing overhead and improving scalability on parallel architectures. The talk concludes with perspectives on the integration of machine learning techniques—such as graph neural networks—for the data-driven construction of coarse basis functions.

Numerical experiments demonstrate the effectiveness of these advances in multiscale and multiphysics simulations, with applications in computational fluid dynamics, land ice modeling, and highly heterogeneous domains.

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A robust two-level restricted additive Schwarz preconditioner based on multiscale spectral generalized FEM for high-Péclet number heterogeneous convection–diffusion

Lukas Holbach

Heidelberg University

We introduce a two-level restricted additive Schwarz (RAS) preconditioner for heterogeneous convection–diffusion equations at high Péclet numbers. Our construction builds on the multiscale spectral generalized finite element method (MS-GFEM), wherein the coarse space is spanned by locally optimal basis functions obtained from local generalized eigenproblems on harmonic spaces. By formulating MS-GFEM as a RAS-type iteration, we employ it as a preconditioner within GMRES. Due to an exponential convergence property of the underlying MS-GFEM, which is independent of the fine-mesh resolution, only a few GMRES iterations are sufficient for accurate solution even when using low-dimensional coarse spaces.

Through extensive numerical experiments on test cases with high-contrast diffusion and non-divergence-free, rotating velocity fields (yielding indefinite operators), we demonstrate:
1. Robustness with respect to the grid Péclet number and the number of subdomains (scaling up to 10^5 subdomains), with coarse-space dimensions that remain small as Péclet numbers increase.
2. Arbitrary convergence rates of preconditioned GMRES by adapting the coarse space and oversampling size.
3. Effectiveness in the vanishing-diffusion limit (formally infinite Péclet number).

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Multicontinuum homogenization for coupled flow and transport equations

Jian Huang

Xiangtan University, China

In this talk, we present the derivation of a multicontinuum model for the coupled flow and transport equations by applying multicontinuum homogenization. We perform the multicontinuum expansion for both flow and transport solutions and formulate novel coupled constraint cell problems to capture the multiscale property, where oversampled regions are utilized to avoid boundary effects. Assuming the smoothness of macroscopic variables, we obtain a multicontinuum system composed of macroscopic elliptic equations and convection-diffusion-reaction equations with homogenized effective properties. Finally, we present numerical results for various coefficient fields and boundary conditions to validate our proposed algorithm.

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High-permeability focusing coarse spaces for overlapping Schwarz preconditioners in high-contrast time-dependent PDEs

Wing Tat Leung

City University of Hong Kong

In this talk, we present a novel multiscale coarse grid space designed for preconditioning high-contrast time-dependent partial differential equations, with a particular focus on parabolic problems such as the Darcy flow. The key innovation lies in constructing a low-dimensional coarse space that targets only the high-permeability regions of the medium, thereby significantly reducing the computational cost while maintaining robustness.

We propose an efficient construction of the coarse space using a relaxed constraint energy minimization framework, which allows for the iterative refinement of basis functions. The resulting space, tailored to capture the dominant multiscale features in high-permeability regions, leads to contrast-independent condition numbers in overlapping domain decomposition methods. Our method enables stable and fast convergence of implicit solvers for time-dependent problems, even in the presence of large parameter variations.

We further analyze a simplified construction for coarse spaces when the temporal discretization step is aligned with the coarse mesh size and demonstrate how a hybrid space combining multiscale and standard finite element components can generalize the framework to more practical scenarios.

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An adaptive BDDC method using vertex-based generalized eigenvalue problems

Olof Widlund

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Domain decomposition algorithms, such as the BDDC methods, are used to solve very large systems of algebraic equations which arise in solving elliptic finite element problems. An adaptive BDDC (Balancing Domain Decomposition by Constraints) method is developed, using a generalized eigenvalue problem defined on each of a subset of a partition of the interface between the subdomains, in order to find effective coarse, primal spaces. The partition is vertex-based and provide a relatively small number of generalized eigenvalue problems for an adaptive BDDC method taking advantage of the fact that the vertices are shared by more subdomains than subdomain edges and faces. In contrast to earlier adaptive BDDC algorithms, these generalized eigenvalue problems are not associated with nodal equivalence classes and have the advantage of needing only one kind of generalized eigenvalue problems. These algorithms always satisfy the null space condition, which is necessary for a convergence rate that does not deteriorate when the number of subdomains increases. The main idea originates from work on the Reduced Adaptive Generalized Dryja-Smith-Widlund – RAGDSW – methods for which global, coarse basis functions are computed adaptively, and differently, for overlapping Schwarz algorithms. A strong theoretical result is provided in terms of a tolerance parameter chosen in the development of the global coarse space.

Joint work with H.H. Kim.

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MS23 – New advances in parallel-in-time methods

Organizers: Bérangère Delourme, Laurence Halpern, Felix Kwok, Julien Salomon

Time-parallelization methods have experienced a major boom in the last two decades. Their level of maturity means they can now be applied to real-life problems and combined with other types of algorithm, such as data assimilation or control. The aim of this mini-symposium is to present the latest methodological and applicative advances in this class of methods.

List of Speakers

- A. Arnoult (Université Sorbonne Paris Nord) – *Coupling parareal with optimized Schwarz waveform relaxation for the Oseen equations*.
- B. Delourme (Université Sorbonne Paris Nord) – *Optimized Schwarz methods for a one dimensional transport control problem*.
- R. D. Haynes (Memorial University of Newfoundland) – *Partitioning and phase changes*.
- S. Hirstoaga (Inria Paris) – *Parareal simulations for systems arising in plasma physics*.
- Z. Miao (Northwestern Polytechnical University) – *A hybrid parareal-WR-MOR approach for long-term building thermal simulation with switching control*.
- L. Perrin (Universität Konstanz) – *A ParaExp strategy for data assimilation*.
- J. Rosemeier (Freie Universität Berlin) – *Multilevel parareal methods and standard form transformations for weakly nonlinear problems*.
- B. Song (Northwestern Polytechnical University) – *Time-parallel methods for time-periodic parabolic optimal control problems*.

Coupling parareal with optimized Schwarz waveform relaxation for the Oseen equations

Arthur Arnoult

Université Sorbonne Paris Nord

In this talk, we present a numerical approach that combines a domain decomposition method for spatial parallelization with a time-parallel algorithm for solving the Oseen problem, which models incompressible viscous flows at usually low Reynolds numbers. More specifically, we propose coupling the Optimized Schwarz Waveform Relaxation (OSWR) algorithm with the Parareal method.

The Parareal algorithm consists in dividing the global time window into sub-windows. Initial (possibly arbitrary) conditions are provided at the start of each sub-window, and a parallel-in-time resolution is performed independently on each of them. A correction step is then applied to update the initial conditions, and the process is iterated until convergence.

In our approach, we use OSWR to solve the space-time subproblems within each Parareal time window, introducing a second level of parallelism. This hybrid strategy uses the complementary efficiency of OSWR and Parareal to significantly reduce computation time. Indeed, only a few iterations of OSWR are required within each Parareal iteration, as convergence is progressively achieved across Parareal iterations.

In this talk, we will detail the mathematical foundations of the coupled algorithm, its implementation, and its numerical performance on representative test cases. The results highlight the computational gains achieved compared to OSWR alone. This study opens new perspectives to efficiently solve the Navier–Stokes equations in a massively parallel framework.

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Optimized Schwarz methods for a one dimensional transport control problem

Bérangère Delourme

Université Sorbonne Paris Nord

We study optimized Schwarz domain decomposition methods in time for the control of the 1D transport equation. In the case of an internal control over the whole domain, the optimization problem can be transformed into a system of two coupled PDEs. We then apply the time-domain decomposition strategy on this PDE system as well as on its discretized counterpart. We show that continuous and discrete optimal parameters are different. We pay particular attention to the study of the preconditioned GMRES method. We illustrate our results by numerical examples.

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Partitioning and phase changes

Ronald D. Haynes

Memorial University of Newfoundland

In this talk we will review recent work on space-time parallelism for the one-phase Stefan problem. After reviewing the physics of liquid-solid phase change, first considered by Stefan in 1889, we will propose Schwarz waveform relaxation and parareal solution approaches. Numerical and (up to the minute) theoretical results will be presented.

Joint work with M.J. Gander.

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Parareal simulations for systems arising in plasma physics

Sever Hirstoaga

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Solving the Vlasov-Poisson system with multiple time scales is a challenging problem in plasma physics. The aim of the present talk is to describe our ongoing efforts to develop an efficient and robust numerical method with the help of the Parareal algorithm. First, we derive reduced models obtained from two-scale asymptotic expansions. These models approximate the original Vlasov-Poisson model at a low computational cost. Second, we use the reduced models for the coarse solving in the Parareal approach. Applications to problems in plasma physics underline the efficiency of the strategy.

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A hybrid parareal-WR-MOR approach for long-term building thermal simulation with switching control

Zhen Miao

School of Mathematics and Statistics, Northwestern Polytechnical University

We propose a hybrid space-time acceleration approach that integrates the parareal algorithm, waveform relaxation (WR), and model order reduction (MOR) for the computational efficiency challenges in dynamic thermal simulations with switching control of building environments. The parareal algorithm is leveraged to enhance computational efficiency for long-term simulations of switched systems. We establish an event-triggered adaptive reduced-order model switching process based on orthogonal function basis, which is embedded into the coarse propagator of the parareal algorithm to reduce the parallel-in-time iteration costs. Moreover, decoupling complex heat transfer processes (e.g., radiation and convection) is achieved via WR to streamline iterative computations. The combined effect of Parareal-WR-MOR enables rapid response in switching control scenarios. For a dual-zone switching control problem, we validate the time-windowed reduced-order model transfer strategy and demonstrate the algorithm's speedup.

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A ParaExp strategy for data assimilation

Lucas Perrin

Universität Konstanz

This talk presents a strategy to accelerate data assimilation by coupling a Luenberger observer—designed to operate over unbounded time intervals—with the time-parallel Paraexp algorithm, which is restricted to finite horizons. The key contribution lies in bridging these two frameworks without degrading the convergence properties of the observer. The method builds on an approximation of the matrix exponential, allowing for an efficient decomposition of the assimilation process into independent time slices. Numerical experiments on hyperbolic test cases, including a one-dimensional linearized water waves equation, highlight both the stability and performance of the proposed approach. As a proof of concept, this work opens the door to integrating simple observer-based assimilation with parallel-in-time techniques in a consistent and effective manner.

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Multilevel parareal methods and standard form transformations for weakly nonlinear problems

Juliane Rosemeier

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The Parareal algorithm is a well-established parallel-in-time method, promising for multiscale oscillatory problems. In this talk, I will present a multilevel extension of the two-level Parareal method with exponential transformation and averaging. The two-level versions were introduced by Peddle, Haut, and Wingate [SIAM J. Sci. Comput., 2019] and Haut and Wingate [SIAM J. Sci. Comput., 2014], whereas the multilevel approach was first established in Rosemeier, Haut, and Wingate [SIAM J. Sci. Comput., 2024]. This multilevel approach generalizes the method to arbitrarily many levels, each with its own averaging window. This flexibility makes the method particularly promising for weakly nonlinear problems with fast oscillations, where multiple interacting scales must be resolved efficiently. Additionally, I will outline ongoing work on a new transformation into standard form, which is related to WKB methods. This transformation can possibly be used to construct new coarse propagators by leveraging structural properties of highly oscillatory problems. While this aspect is still under development, I will discuss preliminary insights.

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Time-parallel methods for time-periodic parabolic optimal control problems

Bo Song

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Optimal control problems governed by the time-periodic parabolic problem arise from many different kinds of engineering problems, such as eddy current problems and power generating kite systems. Usually, by using the Lagrange multiplier, the original problem is converted to the coupled forward and backward system with special time-periodic conditions. However, such system is very difficult to solve numerically directly because of this special time-periodic structure. In this talk, several time-parallel methods are constructed and analyzed. For this special coupled system, the Dirichlet-Neumann/Neumann-Dirichlet and Parareal algorithms are designed, and the convergence analysis of the proposed algorithms is also provided. Finally, several numerical experiments illustrate the effectiveness of the proposed time-parallel algorithms.

Joint work with Y.-L. Jiang.

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MS24 – Preconditioning strategies for saddle point problems

Organizers: Pablo Brubeck Martinez, Umberto Zerbinati

Problems involving linear constraints are ubiquitous in applications, including constrained optimisation and mixed formulations for incompressible flow, electromagnetism, and coupled multiphysics problems, among others. Ensuring well-posedness and stability of discretizations for these problems is a fundamental challenge. Structure-preserving discretisation that satisfy the constraints are also highly desirable and require sophisticated approaches. The design and analysis of robust and scalable solvers for these problems inherit such challenges. This minisymposium focuses on efficient iterative solution techniques for such linear systems with saddle-point structure.

Topics of interest include multigrid and domain decomposition methods, augmented Lagrangian techniques, and novel preconditioning techniques. Special attention will be given to finite element exterior calculus (FEEC) and other approaches that exploit problem structure to design scalable and efficient solvers. Contributions addressing both theoretical developments and practical applications in computational science and engineering are particularly welcome.

The goal of this minisymposium is to bring together researchers from numerical analysis, scientific computing, and applied mathematics to discuss recent advances, foster interdisciplinary collaboration, and inspire new approaches to tackling these challenging problems.

List of Speakers

- E. Bonetti (TU Wien) – *Preconditioning for the Einstein-Bianchi equations*.
- P. Brubeck (University of Oxford) – *High-order and sparsity-promoting Stokes elements*.
- F. Credali (King Abdullah University of Science and Technology, Saudi Arabia) – *Stability and conditioning of a fictitious domain formulation for fluid-structure interaction problems*.
- M. Feder (Università di Pisa) – *Optimal and scalable augmented Lagrangian preconditioners for fictitious domain problems*.
- K. Knook (University of Oxford) – *Preconditioners for multicomponent Flows*.
- L. Saßmannshausen (University of Cologne) – *Monolithic and block overlapping Schwarz preconditioners for the incompressible Navier-Stokes equations*.
- J. Schoeberl (TU Wien) – *Building blocks for preconditioners in NGSolve*.
- U. Zerbinati (University of Oxford) – *Preconditioned normal equations for solving discretised partial differential equations*.

Preconditioning for the Einstein-Bianchi equations

Edoardo Bonetti

TU Wien

Over the past decade, the study of the de Rham complex has played a central role in the development of robust preconditioning strategies for numerical PDEs. More recently, the emergence of distributional 2-complexes has enriched this framework, offering new insights into the structure and discretization of linear, higher-order differential operators.

After briefly revisiting the concept of 2-complexes in the setting of Sobolev spaces, we focus on a specific 1-(sub)complex: the distributional Hessian complex. This complex naturally arises in the 3+1 formalism of linearized general relativity, where tensor fields must satisfy multiple simultaneous constraints—such as symmetry, tracelessness, and/or divergence-freeness. A central challenge lies in selecting suitable function spaces, particularly when the associated differential operators act in a distributional sense. Ensuring compatibility often requires the vanishing of certain undesirable ("bad") terms to preserve closedness and exactness.

We identify and analyze the null spaces of the Hessian complex at both the continuous and discrete levels. This characterization is fundamental to the construction of structure-preserving preconditioners that reflect the underlying differential and algebraic structure of the system.

As a practical application, we develop an additive Schwarz preconditioner (ASM) for the discrete extended H(cc)-space and integrate it into a Locally Optimal Block Preconditioned Conjugate Gradient (LOBPCG) method. This solver is applied to the Einstein–Bianchi eigenvalue problem, a challenging spectral formulation not easily addressed using standard matrix-valued discretizations.

The implementation is carried out in NGSolve, using its user-friendly add-on interface for rapid development and experimentation.

This work highlights the potential of hypercomplex-based preconditioning in developing efficient solvers for systems governed by intricate physical and mathematical constraints.

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High-order and sparsity-promoting Stokes elements

Pablo Brubeck

University of Oxford

One of the long-standing challenges of numerical analysis is the efficient and stable solution of incompressible flow problems (e.g. Stokes). It is fairly non-trivial to design a discretization that yields a well-posed (invertible) linear saddle-point problem. Additionally requiring that the discrete solution preserves the divergence-free constraint introduces further difficulty. In this talk, we present new finite elements for incompressible flow using high-order piecewise polynomials spaces. These elements exploit certain orthogonality relations to reduce the computational cost and storage of augmented Lagrangian preconditioners. We achieve a robust and scalable solver by combining this high-order element with a domain decomposition method, and a lower-order element as the coarse space. We illustrate our solver with numerical examples in Firedrake.

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Stability and conditioning of a fictitious domain formulation for fluid-structure interaction problems

Fabio Credali

King Abdullah University of Science and Technology, Saudi Arabia

We consider a fictitious domain formulation for fluid-structure interaction problems [1] where an incompressible elastic body is immersed in an incompressible Newtonian fluid. Fluid and solid equations are solved on two independent meshes and a distributed Lagrange multiplier is employed to enforce the kinematic constraints.

We discuss implementation details [2] and stability results, with particular focus on stability in presence of small intersection cells, condition number [3] and challenging in designing an effective parallel solver [4].

Joint work with D. Boffi and L. Gastaldi.

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Optimal and scalable augmented Lagrangian preconditioners for fictitious domain problems

Marco Feder

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We present augmented Lagrangian-based preconditioners for efficiently solving linear systems with block two-by-two and three-by-three structures arising in fictitious domain problems and finite element discretizations of immersed boundary methods. We consider two representative cases—the Poisson and Stokes fictitious domain problems—to illustrate the performance of these preconditioners when used with flexible GMRES.

We provide a detailed spectral analysis of the proposed preconditioners, deriving lower and upper bounds for the eigenvalues of the preconditioned matrix, showing their independence with respect to discretization parameters, and discuss the eigenvalue distribution when inexact versions of the preconditioners are employed.

The robustness and efficiency of the proposed methods are validated through extensive numerical experiments in both two and three dimensions, involving various immersed geometries. We further extend the approach to elliptic interface problems (FD-DLM), showing that a modified—yet significantly cheaper—variant of the ideal augmented Lagrangian preconditioner yields an efficient tool for large-scale problems with potentially large coefficient jumps.

Finally, we highlight relevant computational aspects related to the memory-distributed implementation of these methodologies, allowing independent mesh generation while requiring efficient treatment of mesh data. Our parallel implementation is based on the C++ Finite Element library DEAL.II.

Joint work with M. Benzi, L. Heltai, and F. Mugnaioni.

References

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Preconditioners for multicomponent flows

Kars Knook

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In this talk, a Braess-Sarazin preconditioner is presented for the Onsager-Stefan-Maxwell (OSM) equations, which is a general set of equations describing multicomponent diffusion problems. The preconditioner will be applied to a variety of multicomponent flows with non ideal mixing, thermal, pressure, electrochemical effects. The Braess-Sarazin preconditioning strategy can be adapted to many problems posed in the same Sobolev spaces as the OSM equations, $H(rmdiv)$-L^2, and additional fields can straightforwardly be incorporated.

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Monolithic and block overlapping Schwarz preconditioners for the incompressible Navier-Stokes equations

Lea Saßmannshausen

University of Cologne

Different monolithic preconditioning techniques for the incompressible Navier-Stokes equations are introduced and compared to a selection of block preconditioners. In particular, two-level additive overlapping Schwarz methods are used to set up monolithic preconditioners and to approximate the inverses that arise in the block preconditioners. To construct the second level, GDSW-type (generalized Dryja–Smith–Wildund) coarse spaces are used.

These highly-scalable, parallel GDSW preconditioners have been implemented in the solver framework FROSch (Fast and Robust Overlapping Schwarz), which is part of the software library Trilinos.

Monolithic preconditioners are robust because they account for the coupling terms in the system matrix on both levels, that is, in the local and coarse problems.

In comparison, block preconditioners, mostly based on block-diagonal and block-triangular preconditioners, such as the PCD (pressure convection-diffusion) and SIMPLE (semi-implicit method for pressure linked equations) preconditioners, often yield higher iteration counts while having a lower setup cost compared to monolithic approaches.

In this talk, the parallel performance of the different preconditioning methods for incompressible fluid flow problems is investigated and compared using a finite element implementation based on the FEDDLib (Finite Element and Domain Decomposition Library) and the overlapping Schwarz preconditioners from the Trilinos package FROSch. Furthermore, the robustness of these methods is tested for a range of Reynolds numbers with respect to a realistic problem setting.

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Building blocks for preconditioners in NGSolve

Joachim Schoeberl

TU Wien

In this talk, we present building blocks to construct solvers for saddle point problems in NGSolve.

We are mainly interested in finite element discretizations stemming from H(div)-based discetizatio for the Stokes or Navier-Stokes system, such as hybrid discontinuous Galerkin methods, or the mixed MCS method.

We present the combination of block-smoothers and multigrid grid-transfer operators for these discretizations, and discuss robustness with respect to Augmented Lagrangian stabilization parameters.

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Preconditioned normal equations for solving discretised partial differential equations

Umberto Zerbinati

University of Oxford

This paper explores preconditioning the normal equation for non-symmetric square linear systems arising from PDE discretization, focusing on methods like CGNE and LSQR. The concept of "normal" preconditioning is introduced and a strategy to construct preconditioners studying the associated "normal" PDE is presented. Numerical experiments on convection-diffusion problems demonstrate the effectiveness of this approach in achieving fast and stable convergence.

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MS25 – Solution techniques for nonstandard approximations: theory and applications

Organizers: Blanca Ayuso de Dios, Susanne C. Brenner

The aim of the session is to bring together experts who are active in the construction and analysis of solution techniques for non-standard approximations of partial differential equations (PDEs). This includes the invention of novel discretizations methods; the development of solution strategies such as domain decomposition, multilevel and/or randomized methods, as well as the design of adaptivity and dimension reduction techniques.

The devising of novel non standard discretizations for complex partial differential equations (PDEs) is a fascinating field of research. In some instances it is essential to build structure preserving discretizations, while in others the complexity or high dimensionality of the problem itself precludes the use of any conventional discretizations. Among the advanced discretizations methods, one finds various discontinuous Galerkin methods, nonconforming approximations, boundary elements methods, kernel methods, reduced order approximation, to mention a few.

This mini-symposia will bring together experts in the different topics in the field to facilitate the discussion in identifying common points in the design of solution techniques for non-standard methods and to promote collaboration. Sample topics include the design, the theoretical analysis and issues related to the implementation and applications of the various solution techniques.

List of Speakers

- A. Alonso Rodriguez (University of Trento) – *Tree-cotree decomposition of high order Whitney finite elements*.
- M. Bauer (University of Bayreuth) – *Block-adaptively generated H-matrices for computations in linear elasticity*.
- I. Bioli (Università di Pavia) – *Preconditioning PINNs: a randomized approach to natural gradient descent*.
- X. Feng (The University of Tennessee) – *Efficient numerical methods and solvers for parameter-dependent and random PDEs*.
- J. Garay (University of Augsburg) – *Spectral localized orthogonal decomposition method for multi-scale elliptic PDEs with high-contrast channels*.
- R. Hiptmair (ETH Zurich) – *Inherited stability*.
- S. Lee (Tufts University) – *Arbitrary-dimensional monotone schemes for the spatio-temporal transport equation*.
- T. Mao (KAUST) – *Integral representations of Sobolev spaces via ReLU^k activation function and optimal error estimates for linearized networks*.
- C. Rodrigo (University of Zaragoza) – *A decoupled solver for a novel stabilization scheme for Biot's model*.
- A. Veeser (Università degli Studi di Milano) – *Strictly equivalent a posteriori error estimators for quasi-optimal nonconforming methods*.

- H. Yang (IGPM, RWTH Aachen) – *Sparse and low-rank approximations of parametric PDEs: the best of both worlds.*
- Y. Yu (Guangxi University) – *Efficient domain decomposition methods for solving Helmholtz equation with random refractive index.*

Tree-cotree decomposition of high order Whitney finite elements

Ana Alonso Rodriguez

University of Trento

The tree-cotree decomposition is a technique used in the finite element approximation of electromagnetic problems to eliminate the kernel of the curl operator from the discrete curl-conforming space. The key point in this method is the identification of degrees of freedom for edge and nodal finite element spaces such that the matrix of the gradient operator is the all-node incidence matrix of a directed graph. This is straightforward for low order finite elements and it can be extended in a very natural way to the high order case using a particular set of degrees of freedom, the so-called weights, that have a clear geometric localization on the mesh. The classical degrees of freedom for high order Whitney finite elements are moments that have not this direct geometrical localization. A particular isomorphism between weights and moments that preserve the matrix of the gradient operator allows to extend this kind of decomposition to the canonical basis for moments.

Joint work with J. Camaño, E. De Los Santos, and F. Rapetti.

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Block-adaptively generated H-matrices for computations in linear elasticity

Maximilian Bauer

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The equations of linear elasticity are formulated as boundary integral equations and solved using the boundary element method (BEM). Since this approach leads to a fully populated system matrix, the computational cost of BEM is generally quite high. In order to address this issue, a possible strategy is to use algorithms based on hierarchical matrices (H-matrices) and the adaptive cross approximation.

This talk introduces an additional level of adaptivity that is built on top of the previously mentioned techniques. The proposed methods rely on error estimators and refinement techniques known from adaptivity, but are not used here to improve the mesh. It is rather to successively increase the accuracy of the H-matrix approximation with the help of the adaptive elements. These new techniques are applied to both, the efficient solution of the Lamé equations and to the multiplication with given data.

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Preconditioning PINNs: a randomized approach to natural gradient descent

Ivan Bioli

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Recent advancements in Scientific Machine Learning (SciML) optimizers highlight the critical role of preconditioning in accelerating convergence during training. While this concept mirrors its use in traditional iterative methods from Numerical Linear Algebra, adapting it to Neural Network presents challenges due to their inherent non-linearity and non-locality.

A recently proposed preconditioner in the Natural Gradient Descent (NGD) method [1, 2] significantly reduces the number of iterations required for convergence. However, its computational cost scales cubically with the number of network parameters, making it impractical for large-scale problems. To address this, we exploit the empirically observed low-rank structure of the NGD preconditioner and develop a matrix-free implementation that leverages Randomized Numerical Linear Algebra (RandNLA) techniques to accelerate its application.

Numerical experiments with Physics-Informed Neural Networks (PINNs) and Variational PINNs (VPINNs) demonstrate that preconditioning significantly improves convergence rates, while randomization effectively reduces the associated computational overhead. These findings point to a promising direction for optimizing SciML workflows.

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Efficient numerical methods and solvers for parameter-dependent and random PDEs

Xiaobing Feng

The University of Tennessee, USA

In this talk, I will present a solver-guided framework for addressing parameter-dependent and random partial differential equation (PDE) problems. The main idea is to effectively combine an iterative solver with either a multi-mode Monte Carlo method for random PDEs or an ensemble strategy for parameter-dependent problems. A key advantage of this framework is that it requires solving only a fixed deterministic problem at each iteration. This allows for the efficient use of techniques such as direct solvers or block Gauss-Seidel methods, resulting in a more computationally effective approach than brute-force methods commonly used for such problems. I will illustrate the framework through detailed applications to random wave equations, which model wave scattering in random media—and parameter-dependent convection-diffusion equations. To highlight the framework's effectiveness, I will also present convergence analysis and numerical experiments that demonstrate its potential advantages.

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Spectral localized orthogonal decomposition method for multi-scale elliptic PDEs with high-contrast channels

Jose Garay

University of Augsburg

The Localized Orthogonal Decomposition (LOD) method is a numerical homogenization method originally designed to solve multiscale diffusion-type elliptic partial differential equations (PDEs) with rough coefficients. It is based on the use of a generalized finite element space whose basis functions incorporate relevant microscopic information. This improved basis allows solving a much smaller problem to produce an approximate solution with an accuracy comparable to the expensive standard finite element method (FEM) (which typically requires a very fine mesh to resolve all scales of the problem). One of the desired aspects of the LOD method is the exponential decay of the basis functions, which results in basis functions with localized supports and makes their computation fast and inexpensive. However, it is known that for diffusion coefficients containing high-contrast channels, the exponential decay rate of standard LOD basis functions is slowed down due to a contrast-dependent factor, requiring basis functions with significantly large supports to obtain optimal error convergence rates. In this talk, we present a new variant of the LOD method that introduces a novel replacement of the quasi-interpolation operator (originally employed in the construction of the basis corrector space) to enrich the corrector space with local spectral information, which leads to faster decaying basis functions that preserve optimal error convergence rates. Numerical experiments are also provided to illustrate our theoretical findings.

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Inherited stability

Ralf Hiptmair

ETH Zurich

Linear second-order boundary value problems with constant coefficients can often be recast as variational boundary integral equations (BIEs), which are set in "energy trace spaces". To solve the BIEs numerically we can apply Galerkin finite-element discretization, an approach falling into the class of boundary-element methods (BEM). For the h-version of the BEM the resulting linear systems of equations have to be solved iteratively, because the Galerkin matrix will usually only be available in a data-compressed format; preconditioning becomes a concern, thus.

It has been a successful policy in the mathematical and numerical analysis of BIEs to tackle them through the underlying "volume" variational problem. We harness this idea for the analysis of the stability, with respect to the BIE-induced norms, of decompositions of those finite-element spaces on the boundary known as boundary-element spaces. Taking the cue from earlier work by P. Oswald [3] we develop an abstract framework addressing the preservation of stability of decompositions under the action of a trace operator. We apply it to study both multilevel splittings and domain decomposition-type splittings of boundary element spaces for first-kind BIEs on closed and open boundaries (screens).

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Arbitrary-dimensional monotone schemes for the spatio-temporal transport equation

Seulip Lee

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In this talk, we present arbitrary-dimensional monotone schemes for solving the spatiotemporal transport equation, formulated as a stationary convection-diffusion problem in a convection-dominated regime. A central challenge in such problems is to ensure numerical stability while accurately capturing sharp solution features and minimizing numerical dissipation. To address this, we focus on the discrete maximum principle (DMP), which ensures stability and prevents non-physical oscillations. The DMP arises from the monotonicity of the scheme, which is achieved by satisfying the M-matrix condition. Building on this foundation, we develop and analyze numerical schemes that enforce these properties through edge-averaged methods. We present both theoretical insights and numerical results to demonstrate the effectiveness of the proposed approach for stationary convection-dominated problems. Finally, we discuss possible extensions to arbitrary-dimensional spatiotemporal problems, as well as generalizations to curl-curl and grad-div systems.

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Integral representations of Sobolev spaces via ReLU^k activation function and optimal error estimates for linearized networks

Tong Mao

KAUST

We will present two main theoretical results concerning shallow neural networks with ReLU^k activation functions. We establish a novel integral representation for Sobolev spaces, showing that every function in $H^{\frac{d+2k+1}{2}}(\Omega)$ can be expressed as an L^2-weighted integral of ReLU^k ridge functions over the unit sphere. This result mirrors the known representation of Barron spaces and highlights a fundamental connection between Sobolev regularity and neural network representations. Moreover, we prove that linearized shallow networks—constructed by fixed inner parameters and optimizing only the linear coefficients—achieve optimal approximation rates in Sobolev spaces.

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A decoupled solver for a novel stabilization scheme for Biot's model

Carmen Rodrigo

University of Zaragoza

The coupling of fluid flow and mechanical deformation within a porous media is studied by Biot's model. For the numerical simulation of this poroelastic problem, there are mainly two ways to deal with the solution of the large sparse systems arising after discretization of the problem. Namely, fully coupled or monolithic methods and iterative coupling methods. In this talk, we present a decoupled method falling in this latter group. The proposed iterative scheme naturally appears by iterating between the flow and mechanics problems from a new stabilized discretization of the model. We demonstrate the parameter-robust convergence of the proposed iterative coupling method, and show the optimality of this method for one-dimensional problems and their good behavior in higher dimensions.

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Strictly equivalent a posteriori error estimators for quasi-optimal nonconforming methods

Andreas Veeser

Università degli Studi di Milano

The talk will present a posteriori error estimators for quasi-optimal nonconforming finite element methods approximating symmetric elliptic problems of second and fourth order. These estimators are defined for all source terms that are admissible to the underlying weak formulations. More importantly, they are equivalent to the error in a strict sense. In particular, their data oscillation part is bounded by the error and, furthermore, can be designed to be bounded by classical data oscillations. The estimators split into local contributions which are computable, except for the data oscillation part. As for other estimators, the later issue arises from the infinite-dimensional nature of general data and, therefore, we advocate to handle it on a case-by-case basis.

The aforementioned properties of the estimators rely on a domain decomposition and suitably discretizing local operators.

The practical use of the estimators will be illustrated for a source term to which classical nonconforming methods cannot be applied.

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Sparse and low-rank approximations of parametric PDEs: the best of both worlds

Huqing Yang

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We introduce a new type of sparse and low-rank approximation for solutions of parametric partial differential equations depending on infinitely many parameters, combining low-rank tensor approximation in a subset of dominant parameters with a sparse polynomial expansion in the remaining variables. This differs from usual low-rank approaches for such problems based on separating all variables, and it addresses in particular classes of elliptic problems - for example, with random diffusion coefficients of short correlation length - where a direct polynomial expansion is inefficient. Based on this approximation format, we propose a convergent adaptive Galerkin solver that uses tensor soft thresholding for rank reduction and refines discretizations based on lower-dimensional projected quantities. Unlike existing adaptive low-rank schemes, we obtain quasi-optimal ranks of all iterates and at the same time optimal convergence rates of spatial discretizations without a coarsening step. The results are illustrated by numerical tests.

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DD 29

Efficient domain decomposition methods for solving Helmholtz equation with random refractive index

Yi Yu

Guangxi University

In this talk, we consider an efficient numerical scheme solving the stochastic Helmholtz equation with random refractive index through sampling-based methods such as Monte Carlo (MC) or quasi-Monte Carlo (QMC). However, these methods typically require a large number of sampling points, and solving the corresponding deterministic problems for each sample is computationally intensive. To address this challenge, we propose a new domain decomposition framework and preconditioners based on approximating the global negative eigenmode of the indefinite system, while for the positive definite component, we use a traditional elliptic preconditioner. The preconditioner is well-posed without any constraints, and the size of the global problem is the number of negative eigenfunctions. Numerical experiments demonstrate the efficacy of the proposed method, especially in stochastic sampling-based methods, as we reuse a single preconditioner across all deterministic solves, significantly reducing computational costs.

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MS26 – Transmission conditions in domain decomposition for steady and evolution problems with applications

Organizers: Martin J. Gander, Yingxiang Xu

All domain decomposition algorithms need transmission conditions in order to function, because subdomain problems need to exchange information in the domain decomposition iteration for convergence to be possible. Classical transmission conditions are of Dirichlet type (e.g. Schwarz methods) and of Dirichlet and Neumann type (e.g. Dirichlet-Neumann, Neumann-Neumann and FETI methods). When domain decomposition methods are used for more difficult tasks than just solving linear systems, like in heterogeneous domain decomposition methods, when solving time dependent and/or nonlinear problems, or in applications with physics that are far beyond simple diffusion problems, like time harmonic wave propagation, it can be advantageous to use more sophisticated transmission conditions between subdomains, in order to obtain robust parallel solvers with rapid convergence. This minisymposium brings researchers working in various fields together to discuss transmission conditions in domain decomposition in a very wide context.

List of Speakers

- M. Al-Khaleel (Khalifa University) – *Neumann–Neumann waveform relaxation approach with applications to time-fractional circuit models*.
- B. Chaudet-Dumas (University of Applied Sciences and Arts of Western Switzerland) – *A sweeping domain decomposition method for elliptic problems*.
- M. Genseberger (Deltares) – *Exploring different domain decomposition approaches for enhanced modelling of real-life applications in lakes*.
- M. Kern (Inria) – *A posteriori stopping criteria for optimized Schwarz domain decomposition algorithms in mixed formulations*.
- S. Liao (University of Geneva) – *Nonlinear preconditioning for linear complementarity problems*.
- S. Loisel (Heriot-Watt University) – *Efficient solvers for p-Laplace and related problems*.
- B. Mandal (IIT Bhubaneswar) – *Dirichlet-Neumann and Neumann-Neumann Waveform relaxation methods for PDEs with time delay*.
- A. Pogozelskyte (University of Geneva) – *Varying coarse solvers across time-intervals in parareal*.
- V. Schüller (Lund University) – *Analysis of bulk interface conditions for atmosphere-ocean-sea ice coupling*.
- B. Song (Northwestern Polytechnical University) – *Parareal optimized Schwarz waveform relaxation algorithms for the heat equation*.
- Y. Sun (University of Geneva) – *Multigrid methods for the Helmholtz equation with Robin boundary condition*.

- J. Yang (University of Geneva) – *Optimal prolongation and restriction operators for space-time multigrid methods*.

Neumann–Neumann waveform relaxation approach with applications to time-fractional circuit models

Mohammad Al-Khaleel

Khalifa University

The classical waveform relaxation (cWR) techniques work by breaking down large systems of ordinary differential equations (ODEs) into smaller subsystems, which are then solved iteratively using Jacobi or Gauss–Seidel schemes. However, identifying an effective decomposition is often challenging, and for tightly coupled systems, cWR methods tend to exhibit slow and inconsistent convergence. In contrast, longitudinal WR approaches, such as the Robin WR (RWR) and the Neumann–Neumann WR (NN-WR) methods, offer benefits like straightforward partitioning and more uniform convergence behavior. While the RWR method has been widely examined in recent years, the NN-WR method is relatively new and remains underexplored. In this study, we analyze the convergence of the NN-WR method when applied to time-fractional RC circuit models, with a focus on optimizing the involved parameter, namely β. This investigation effectively serves as an exploration of the NN-WR method at the semi-discrete level for time-fractional partial differential equations. Comprehensive numerical experiments are conducted to evaluate convergence behavior, computational efficiency (CPU time), and the method's asymptotic response to various problem and discretization parameters, across both two and multi-subcircuit configurations. Our research demonstrated that the NN-WR method can significantly outperform the RWR method in terms of convergence speed, given an appropriate choice of its key parameter β.

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A sweeping domain decomposition method for elliptic problems

Bastien Chaudet-Dumas

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In this presentation, we introduce a new domain decomposition method based on an analytic decomposition of the solution. For a given set of subdomains, the approach relies on a corresponding decomposition of the solution into pseudo-even/odd components (one for each dimension variable). Thanks to this decomposition, we are able to compute exactly the solution in boundary subdomains, and then we can propagate the information step by step towards the center of the domain. For a simple linear elliptic problem, we prove that the method converges in a finite number of iterations, both in 1D and 2D for generic sets of regular subdomains. Finally, we illustrate our results with numerical experiments.

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Exploring different domain decomposition approaches for enhanced modelling of real-life applications in lakes

Menno Genseberger

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Currently, several practical developments related to different societal aspects are being investigated for Lake IJssel and Lake Marken in the Netherlands. To facilitate this with integral impact assessments, there is a need of hydrodynamic and water quality models that have enough quality and resolution and are fast enough on modern hardware. Many of those developments consist of a local measure in a small part of the lake that may affect the natural lake system on the global scale. This poses specific requirements for a model approach. On one hand, the local measure has to be schematized with enough resolution in a small part of the lake. On the other hand, the model needs to represent enough the global system behavior further away.

A typical and very actual example is the extension of Marker Wadden, a collection of archipelago type of islands, in the east of Lake Marken. Although there is a transition to the Delft3D FM software [3], practical model application for this example is still with the Delft3D(4) software [4]. For that purpose, the applied silt model of Lake Marken [2] uses coupled flow wave computations. Flow computations are with a shallow water solver from Delft3D(4), wave computation are with SWAN [6]. By using online nesting on modern hardware with many cores within shared memory, the wave part is not a computational bottleneck anymore. Therefore the question remains for the flow part: how to have enough local resolution around the measure with enough global system behavior in the model while computations are fast enough? The presentation illustrates how current possibilities with domain decomposition are explored for this question. One approach in Delft3D(4) works relatively fast on the solver level in shared memory. In Delft3D FM unstructured meshes may be beneficial for smooth transitions in spatial refinements. Also, a third shallow water solver from SIMONA [5] is considered. Delft3D(4) and SIMONA use the same ADI time integration method on structured meshes. However, they differ in the domain decomposition approaches for parallel computing and modelling flexibility. Here SIMONA enables fast and efficient computing by optimized interface (or transmission) conditions [1]. Discussion is how to combine these approaches for enhanced modelling capabilities.

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A posteriori stopping criteria for optimized Schwarz domain decomposition algorithms in mixed formulations

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This paper develops a posteriori estimates for domain decomposition methods with optimized Robin transmission conditions on the interface between subdomains. We choose to demonstrate the methodology for mixed formulations, with a lowest-order Raviart–Thomas–Nédélec discretization, often used for heterogeneous and anisotropic porous media diffusion problems.

A posterior estimates are computed based on the reconstruction of both H^1 conforming scalar potentials and an $H(\mathrm{div})$ conforming and locally equilibrated vector flux. Because of the Robin transmission conditions, neither the scalar nor the vector unknowns are continuous throughout the iterations. The reconstruction of the flux first solves a coarse balancing problems, then adds a correction by solving Neumann problems in narrow bands across the interface. Two potential are reconstructed: the first one is globally continuous thanks to a standard averaging procedure, while the second one uses weights on the interface that vary during the iterations so as to separate the DD error from the discretization error.

Our estimators thus allow to distinguish the spatial discretization and the domain decomposition error components. We propose an adaptive domain decomposition algorithm wherein the iterations are stopped when the domain decomposition error does not affect significantly the overall error.

Two main goals are thus achieved:
- First, a guaranteed bound on the overall error is obtained at each step of the domain decomposition algorithm.
- Second, important savings in terms of the number of domain decomposition iterations can be realized.

Numerical experiments illustrate the efficiency of our estimates and the performance of the adaptive stopping criteria.

Joint work with S. Ali Hassan, C. Japhet , and M. Vohralík.

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Nonlinear preconditioning for linear complementarity problems

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The linear complementarity problem (LCP) arises in many scientific computing and engineering applications, e.g., free boundary problems, contact problems, financial option pricing problems, and market equilibrium problems. These problems are typically large-scale and sparse, making iterative methods particularly suitable due to their low storage requirements and efficient parallel implementation on high–performance computers. In our work, we systematically study various iterative algorithms, such as modulus–based matrix splitting iteration methods, Schwarz methods, projection algorithms, and also propose these methods as nonlinear preconditioners for Newton methods. We illustrate all our results with numerical experiments.

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Efficient solvers for p-Laplace and related problems

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The p-Laplace problem seeks to minimize a nonlinear functional of the form $J(u) = \int_\Omega \|\nabla u\|_2^p + f u \, dx$, where $\Omega \subset \mathbb{R}^d$ is a domain, $p \geq 1$ is a parameter, f is a given forcing, and the unknown $u \in W^{1,p}(\Omega)$ is subject to some Dirichlet boundary conditions. When $p = 2$, this coincides with the usual linear Laplace problem, expressed in energy minimization form (i.e. the Dirichlet principle). When $p \neq 2$, this is a nonlinear problem with applications, e.g. in compressed sensing. The objective functional $J(u)$ is convex in the unknown u so the solution of the p-Laplace problem can be obtained by methods of convex programming. The advantage of this approach is that it produces an algorithm whose global convergence can be analyzed. In this talk, we shall discuss some versions of the barrier method for solving the p-Laplace problem. The basic barrier method converges in $\tilde{O}(\sqrt{n})$ Newton iterations, where the tilde indicates we neglect some polylogarithm. We shall also discuss newer multigrid-based algorithms that converge in $\tilde{O}(1)$ Newton iterations, for finite elements and for spectral elements, provided some regularity conditions are satisfied.

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Dirichlet-Neumann and Neumann-Neumann waveform relaxation methods for PDEs with time delay

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We present a comprehensive theoretical and numerical convergence analysis of two DD-based waveform relaxation algorithms: the Dirichlet-Neumann Waveform Relaxation (DNWR) and Neumann-Neumann Waveform Relaxation (NNWR) methods, applied to a wide class of time-delayed PDEs, including parabolic, hyperbolic, and neutral equations. Time-delayed partial differential equations (PDEs) arise naturally in numerous scientific and engineering domains, including wave propagation, control systems, and biological modeling, where the system's current behavior is influenced by its past states. These methods are examined for both symmetric and asymmetric subdomains, with an emphasis on hyperbolic systems. To support the theoretical conclusions and compare the results with existing Schwarz Waveform Relaxation techniques, numerical examples are provided.

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Varying coarse solvers across time-intervals in parareal

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Parallel-in-time algorithms parallelize partial differential equations by decomposing the problem in time. A prominent example is Parareal, introduced in 2001 by Lions, Maday and Turinici, which uses a coarse operator to transport information between the interfaces of the domains. In this talk, we show that changing the coarse operator based on the considered interface can accelerate the convergence of the method. We give the intuition behind these choices by interpreting the Parareal algorithm as a one-step Runge-Kutta method. Such an interpretation offers many opportunities to further the theoretical understanding we have on Parareal.

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Analysis of bulk interface conditions for atmosphere-ocean-sea ice coupling

Valentina Schüller

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The atmosphere, ocean, and sea ice components in Earth system models are coupled at the sea surface via boundary conditions. In essence, this amounts to coupled heat equations with discontinuous material parameters. However, the problem is special in two ways: First of all, the boundary conditions used in practice, so-called bulk interface conditions, allow for a temperature jump across the interface. Secondly, sea ice acts as a partially isolating layer and affects the boundary conditions seen by the atmosphere and ocean.

Theoretical analysis of this problem is missing, even with simplified models. For this reason, we propose a coupled toy model that describes the heat exchange of a partially ice-covered ocean with the atmosphere. This allows us to analytically derive convergence factors of the corresponding coupling iteration. We compare this to a numerical implementation of the same model using an open-source coupling software for climate applications, ClimaCoupler.jl. Our results show that the convergence behavior with bulk interface conditions is fundamentally different from using standard Dirichlet-Neumann or Robin-Robin interface conditions for conjugate heat transfer.

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Parareal optimized Schwarz waveform relaxation algorithms for the heat equation

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The classical Parareal Schwarz Waveform Relaxation (PSWR) algorithm is a space-time parallel algorithm for the solution of evolution partial differential equations. It is based on a decomposition of the entire domain both in space and in time into smaller space-time subdomains, and then computes by an iteration in parallel on all these small subdomains a better and better approximation of the overall solution. The initial conditions in the subdomains are updated using a Parareal mechanism, while the boundary conditions are updated using Schwarz waveform relaxation (SWR) techniques. However, in the classical PSWR algorithm, the Dirichlet transmission conditions inhibiting the information exchange between subdomains slow down the convergence speed of PSWR. In this talk, we introduce Parareal Optimized Schwarz Waveform Relaxation (POSWR) algorithms both in the continuous and semi-discrete cases, using a class of optimized transmission conditions of Robin type to improve the convergence performance. Convergence factor estimates based on the Laplace transform are provided, and the optimized parameter in the Robin transmission conditions to optimize the convergence behavior of the proposed algorithms is analyzed, when applied to the representative one dimensional heat equation. Finally, several numerical experiments illustrate our theoretical analysis and the performance of the proposed algorithms.

Joint work with Y.-L. Jiang and R.-H. Zhang.

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Multigrid methods for the Helmholtz equation with Robin boundary condition

Yafei Sun

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In the case of Dirichlet boundary conditions, a rigorous Fourier analysis reveals why the basic components of the multigrid method, smoothing and coarse-grid correction, fail to be effective for Helmholtz problems. These difficulties can be addressed in 1D by appropriate modifications, including (1) the use of multi-step smoothers and (2) the introduction of a modified wave number determined by dispersion correction, ultimately leading to a convergent iterative scheme. Since Dirichlet boundary conditions do not necessarily yield a well-posed Helmholtz problem, it is of interest to consider Robin boundary conditions instead. Like it was done recently for domain decomposition methods, in this work, we extend the aforementioned stabilization techniques developed for the Dirichlet case to the Robin case. By studying numerically the contraction factor of a two-gird scheme for the 1D Helmholtz equation, which is discretized by a standard finite-difference scheme, we show that with these improvements the two basic components can also have a good effect on the two-grid method. Additionally, we present several interesting phenomena observed during the numerical investigation. We finally conclude by discussing further research directions, including a possible asymptotic Fourier analysis that we plan to pursue in the near future.

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Optimal prolongation and restriction operators for space-time multigrid methods

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The performance of space-time multigrid methods is influenced not only by the smoother but also by the prolongation and restriction operators, as one can see from numerical results. This observation motivates our investigation to derive optimal prolongation and restriction operators, in the sense of leading to a direct solver, i.e. convergence in a single iteration. This is then followed by introducing approximations to specific components to enhance computational efficiency. By exploiting the structure of the temporal evolution equation, where information propagates only forward in time, and drawing inspiration from cyclic reduction methods for scalar ODEs, we derive exact prolongation and restriction operators. In this context, the transmission of information from the fine to the coarse grid and back can be viewed as a type of transmission condition, which is the core component of our methodology. We analyze the parallel computational complexity associated with various approximation strategies and compare our method with existing space-time multigrid approaches, such as those employing a Block Jacobi smoother and MGRIT. While our method shares similarities with these approaches, it also exhibits significant differences, which will be discussed in detail during the presentation.

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MS27 – Domain decomposition methods and nonlinear preconditioning for nonlinear PDEs

Organizers: Xiao-Chuan Cai, Axel Klawonn, Martin Lanser

Many problems in science and engineering are described by nonlinear partial differential equations (PDEs) and some of them are quite difficult to solve using iterative methods. A common approach to improve at least the linear iterative solvers within, for example, Newton's method is using domain decomposition methods (DDMs). Additionally, one can also reduce the difficulties, or the so-called high nonlinearities, in the nonlinear problems by a nonlinear preconditioning technique which, in some sense, brings the approximate solution closer to the desired solution at least locally. Many efficient nonlinear preconditioners are based on classical linear DDMs and can also be considered as nonlinear DDMs. In this mini-symposium, we present some recent developments of efficient DDMs for nonlinear PDEs with a special focus on nonlinear preconditioning algorithms based on domain decomposition principles. We discuss new efficient and robust linear DDMs used within a nonlinear iterative solver as well as nonlinear DDMs additionally improving the nonlinear convergence.

List of Speakers

- E.M. Ettaouchi (EDF R&D Paris-Saclay) – *Parallel nonlinear Schwarz domain decomposition solvers for two-phase flow in porous media*.
- K. Ho (University of Cologne) – *Parallel scalable monolithic two-level nonlinear Schwarz methods for Navier-Stokes equations with high Reynolds numbers*.
- D. Keyes (KAUST) – *Comparison of Jacobian-preserving nonlinear preconditioning techniques*.
- A. Klawonn (University of Cologne) – *Exploiting machine learning in (nonlinear) two-level Schwarz methods*.
- S. Köhler (Technische Universität Bergakademie Freiberg) – *Multilevel overlapping Schwarz preconditioners for Navier-Stokes problems*.
- M. Lanser (University of Cologne) – *Nonlinear two-level Schwarz methods avoiding global sparse matrix computations*.
- C. Lohmann (TU Dortmund University) – *A multigrid algorithm for discretely divergence-free finite elements solving the three-dimensional incompressible Navier-Stokes equations*.
- A. Marelli (Université Côte d'Azur, INRIA Sophia-Antipolis) – *A nonlinear preconditioning technique for unbalanced nonlinear systems*.

Parallel nonlinear Schwarz domain decomposition solvers for two-phase flow in porous media

El Mehdi Ettaouchi

EDF R&D Paris-Saclay

In the context of radioactive waste storage, the need for advanced parallel nonlinear solvers becomes evident when simulating the surrounding geological environment. These models, which involve two-phase flow equations in porous media, are highly nonlinear and are solved over time frames that can span millions of years, presenting a significant computational challenge. To address this need, innovative nonlinear solvers have been developed and applied, demonstrating their effectiveness in handling complex environmental simulations. These solvers are particularly notable for their use of nonlinear domain decomposition methods, which not only enable efficient parallelization but also significantly improve both nonlinear and linear convergence rates. This improvement is achieved by partitioning the problem into subdomains and solving smaller subproblems within each.

Thanks to an implementation using petsc4py, the solver's capabilities were validated through application to an international benchmark modeling the injection of hydrogen into an initially saturated porous medium. In this presentation, we will showcase the solver's performance on the aforementioned benchmark, highlighting its robustness and rapid convergence. Moreover, the slowdown of information transfer becomes significant when handling a large number of subdomains and negatively affects the outer nonlinear iterations and the inner linear iterations. We shall present a novel second-level strategy that employs an algebraic nonlinear coarse problem introduced at each step to ensure global communication between all subdomains. The acceleration achieved by this two-level approach, both for nonlinear and linear convergence, will be demonstrated on the industrial benchmark problem as well as on other academic test cases. Furthermore, the sequence in which the first-level and second-level operations are applied plays a significant role in enhancing overall performance. The impact of properly ordering these operations on the efficiency of the method will also be discussed in this presentation.

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Parallel scalable monolithic two-level nonlinear Schwarz methods for Navier-Stokes equations with high Reynolds numbers

Kyrill Ho

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When solving nonlinear partial differential equations, the well-known class of linear domain decomposition methods (DDMs) is typically employed as a preconditioner for the Krylov subspace method used to solve the tangential system at each iteration of a Newton-type method.

Nonlinear DDMs offer an alternative to classical Newton-Krylov-DDMs by applying domain decomposition before linearization and solving nonlinear problems both locally and globally. In this sense, they can be interpreted as nonlinear preconditioners to Newton's method and have been shown to improve its nonlinear convergence behavior. Additionally, two-level nonlinear DDM variants hold the potential for excellent scalability. However, these advantages come at the cost of high implementation complexity, which has limited most research to sequential implementations and only a few parallel implementations reached an overall satisfying scalability.

We are developing a highly scalable implementation of one- and two-level nonlinear Schwarz methods based on FROSch, a subpackage of the Trilinos project that implements linear one- and multi-level Schwarz DDMs. In this talk, we describe how our implementation utilizes GDSW-type coarse spaces from the linear Schwarz framework of FROSch in the nonlinear second level of a two-level nonlinear Schwarz method. Furthermore, we present results demonstrating the nonlinear convergence and parallel scalability of our implementation for a selection of nonlinear problems. Specifically, we solve the lid-driven cavity problem at high Reynolds numbers using a monolithic approach and compare the performance of our implementation with that of a standard Newton-Krylov-Schwarz solver. Finally, we discuss key observations from our implementation and performance analysis, focusing on the role of GDSW-type coarse spaces in a nonlinear DDM setting, and explore potential future applications of our approach.

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Comparison of Jacobian-preserving nonlinear preconditioning techniques

David Keyes

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We introduce a novel nonlinear elimination-based inexact Newton algorithm, NEINA, and its simplified variant, SNEINA, that address problems characterized by localized strong nonlinearities. The algorithms employ a subspace correction mechanism through nonlinear elimination that implicitly updates components that most severely hinder global convergence. This process generates a modified direction for the global Newton iteration. We conduct numerical experiments using five different Jacobian-preserving nonlinearly preconditioned inexact Newton algorithms on challenging benchmark problems, establishing NEINA and SNEINA as valuable complements to the growing family of Jacobian-preserving nonlinearly preconditioned inexact Newton algorithms. Applications include flame sheet combustion, lid-driven cavity flow, and backward-facing step flow. The results demonstrate the effectiveness of NEINA and SNEINA in reducing the number of Newton iterations in cases where existing Jacobian-preserving algorithms either converge slowly or fail to converge at all.

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Exploiting machine learning in (nonlinear) two-level Schwarz methods

Axel Klawonn

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The linear convergence rate of classical DDMs deteriorates for coefficient distributions with large contrasts in the coefficient function. To retain the robustness for such problems, the coarse space of the DDM can be enriched by additional coarse basis functions, often obtained by solving local generalized eigenvalue problems. In the context of overlapping Schwarz methods, we consider the AGDSW (adaptive generalized Dryja-Smith-Widlund) coarse space to obtain a robust linear convergence.

Modern nonlinear two-level Schwarz methods, where the global nonlinear problem is decomposed before linearization, have the additional potential to improve the nonlinear convergence and increase the convergence radius of Newton's method. This effect is often even enhanced when an AGDSW coarse space is used within the nonlinear Schwarz approach. Unfortunately, the computation of the AGDSW coarse basis functions is computationally very expensive due to the solution of many local eigenvalue problems. In this talk, we train a surrogate model based on a deep feedforward regression neural network, which directly learns the necessary coarse basis functions themselves. Hence, we can completely replace the computationally most expensive part of the setup, that is, the solution of local eigenvalue problems. As training data for the proposed surrogate model, we use an image representation of the underlying coefficient function such that the model can be trained independently of the underlying finite element discretization. We present results for the linear overlapping Schwarz method as well as the nonlinear Schwarz approach using the learned constraints.

This presentation is based on joint work with Martin Lanser and Janine Weber, University of Cologne, Germany.

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Multilevel overlapping Schwarz preconditioners for Navier-Stokes problems

Stephan Köhler

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Additive overlapping Schwarz Methods are domain decomposition methods for the solution of partial differential equations. A second level, the coarse problem, ensures scalability of these methods. One famous coarse space is the generalized Dryja–Smith–Widlund (GDSW) approach. In [2], monolithic overlapping Schwarz preconditioners for saddle point problems were introduced. We present parallel results for the solution of incompressible fluid problems by the combination of the additive overlapping Schwarz solvers implemented in the fast and robust overlapping Schwarz (FROSch) library, which is part of the Trilinos package ShyLU [4, 3], and the FEATFLOW library [1].

This work is part of the project StrömungsRaum - Novel Exascale-Architectures with Heterogeneous Hardware Components for Computational Fluid Dynamics Simulations is funded by the Bundesministerium für Bildung und Forschung (BMBF).

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Nonlinear two-level Schwarz methods avoiding global sparse matrix computations

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In recent years, nonlinear domain decomposition methods (DDMs) have become more and more popular. They are characterized by a decomposition of the nonlinear problem before linearization - usually with Newton's method. If set up appropriately, nonlinear DDMs show a faster nonlinear convergence and a larger convergence radius than Newton-Krylov methods equipped with a linear DD preconditioner (Newton-Krylov-DDMs). Additionally, nonlinear DDMs tend to increase the local work while reducing the amount of communication in parallel implementations. Therefore, they have the potential to reduce the computing time and improve parallel scalability. Nevertheless, they cannot use modern hardware (as GPUs) more efficiently than classical linear DDMs since they still rely on many computations with distributed and large sparse matrices.

In this talk, we specifically consider nonlinear two-level Schwarz methods with GDSW (Generalized Dryja-Smith-Widlund) coarse spaces. We discuss how they can be used to reduce the amount of global sparse matrix computations. We suggest using alternatives to Newton's method for all global computations and show why similar approaches cannot be used in a classical Newton-Krylov-DDM framework. In other words, we show why using nonlinear DDMs can be beneficial in the context of matrix-free nonlinear preconditioners. We also introduce new coarse spaces based on new nonlinear extensions, which further improve the nonlinear convergence.

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A multigrid algorithm for discretely divergence-free finite elements solving the three-dimensional incompressible Navier-Stokes equations

Christoph Lohmann

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In this talk, we present an efficient multigrid solution technique based on discretely divergence-free finite elements (FE) for the incompressible Navier-Stokes equations in three dimensions. In the proposed approach, each component of the velocity field is approximated by a continuous, piecewise triquadratic FE function, while the discrete pressure variable is represented by a discontinuous, piecewise linear function (so-called Q2-P1 finite element pair). Instead of solving the mixed formulation using traditional Schur complement techniques, we eliminate the pressure variable in an a priori manner by using discretely divergence-free FE functions. This transformation reduces the problem size and removes the saddle point structure, allowing greater flexibility in the design of efficient and/or accurate preconditioners for iterative solution techniques.

This benefit comes at the cost of a singular system matrix due to the fact that the spanning set of discretely divergence-free FE functions is linearly dependent in the first place. To make direct solution techniques applicable, graph-based algorithms are introduced to select a basis from the spanning set.

The presented multigrid algorithm employs a highly-specialized intergrid transfer operator and achieves mesh-independent convergence rates, making it competitive to state-of-the-art monolithic multigrid solvers for a mixed formulation on uniform triangulations. For geometries, where the flow exhibits bifurcations, so-called 'global FE functions' are used to ensure the correct net flux through individual branches.

Various linear and nonlinear test problems, including geometries with different shapes and boundary conditions, illustrate the strengths and limitations of this solution concept for three-dimensional flow simulations.

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A nonlinear preconditioning technique for unbalanced nonlinear systems

Alessandra Marelli

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The numerical discretization of nonlinear partial differential equations, such as the Richards equation, results in large algebraic nonlinear systems of equations. These systems are typically solved using Newton's method, which exhibits quadratic convergence under certain assumptions on the initial point and the residual function. Additionally, under the further conditions of concavity of the residual function and nonnegative inverse of its Jacobian, Newton's method ensures global monotone convergence. Despite these favorable properties, Newton's method can encounter significant computational challenges, leading to a deterioration in performance. In this context, nonlinear preconditioning techniques play a crucial role in enhancing its effectiveness. The presentation focuses on the iterative NlEm (Nonlinear Elimination method), a nonlinear preconditioning technique designed to handle problems with unbalanced nonlinearities that are cases where Newton's method usually struggles. The iterative NlEm directly targets the degrees of freedom responsible for stiff nonlinearities through a local nonlinear elimination process. We provide a presentation of the iterative NlEm algorithm and conduct a convergence analysis for its additive and multiplicative versions, proving their global monotone convergence under two key assumptions: the concavity of the residual function and the M-matrix property of its Jacobian. Next, we apply the iterative NlEm to three case studies concerning the Richards equation, the porous medium equation and the obstacle problem. The numerical experiments on the three applications confirm the improved performance of the iterative NlEm compared to the standard Newton method. Additionally, they demonstrate the robustness of the method with respect to both the problem's physical parameters and the mesh refinement.

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MS28 – Learning-based algorithms and applications

Organizers: Xiao-Chuan Cai, Axel Klawonn, Janine Weber

In this mini-symposium, we discuss some recent developments of machine learning-based algorithms as solvers, preconditioners or as part of preconditioners for solving problems arising from discretizations of partial differential equations or for data-based applications in medical imaging. In particular, we focus on learning techniques in connection with domain decomposition and other subspace iterative methods. Several classes of applications will be considered, including the processing of medical images and the numerical solution of linear and nonlinear partial differential equations.

List of Speakers

- X.-C. Cai (University of Macau) – *A learning-accelerated Newton-Krylov-Schwarz method and applications*.
- S. Pagani (Politecnico di Milano) – *Learning models of disease evolution from sparse data*.
- F. Regazzoni (MOX, Dep. of Mathematics, Politecnico di Milano) – *Learning surrogate solvers of parametrized time-dependent PDEs through latent dynamics networks*.
- J. Weber (University of Cologne) – *Domain decomposition for image classification problems*.

A learning-accelerated Newton-Krylov-Schwarz method and applications

Xiao-Chuan Cai

Department of Mathematics - University of Macau

Newton-Krylov-Schwarz is one of the basic algorithms for solving nonlinear algebraic systems arising from the discretization of nonlinear partial differential equations, and it performs quite well for many different types of problems such as fluid dynamics and solid mechanics with a provable scalability that is close to be linear; i.e., O(n) where n is the number of unknowns. In other words, the numbers of nonlinear (Newton) and linear (Krylov) iterations are both nearly independent of the mesh size and the number of processors. However, in some difficult situations, the big "O" can be large, and therefore the computational time can be large. In this talk, we discuss some learning-based approaches trained using intermediate solutions from the Newton and Krylov iterations to reduce the big "O" and speedup the convergence.

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Learning models of disease evolution from sparse data

Stefano Pagani

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Many diseases are inherently multiscale in space and time, and their clinical manifestations frequently emerge with a significant delay with respect to the onset of the underlying pathophysiological mechanisms. Being able to describe these progressive or even chronic trajectories within a mathematical modeling framework remains an open challenge. Furthermore, even when general parameterized models can be derived, the process of model personalization, that is the identification of subject-specific parameters from clinical data, is often ill-posed due to the sparsity, irregular sampling and incompleteness of available measures.

In this talk, we will introduce a novel scientific machine learning approach that integrates data-driven techniques and dynamical system modeling to capture time complexity. Our hybrid architectures extract patterns from clinical databases, reconstructing trajectories on low-dimensional subspaces using neural ordinary differential equations. These latter are linked to available measures via suitable data-driven or physics-based maps in an end-to-end training strategy. Finally, as additional data is collected, we fine-tune predictions in a data-assimilation setting by conditioning the model with additional variables. We will present some numerical results validating this scientific machine learning approach on real clinical data.

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Learning surrogate solvers of parametrized time-dependent PDEs through latent dynamics networks

Francesco Regazzoni

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Predicting the evolution of systems with spatio-temporal dynamics in response to external stimuli is essential for scientific progress. Traditional equations-based approaches leverage first principles through the numerical approximation of differential equations, thus demanding extensive computational resources. In contrast, data-driven approaches leverage deep learning algorithms to describe system evolution in low-dimensional spaces. We introduce an architecture, termed Latent Dynamics Network, capable of uncovering low-dimensional intrinsic dynamics in potentially non-Markovian systems. Latent Dynamics Networks automatically discover a low-dimensional manifold while learning the system dynamics, eliminating the need for training an auto-encoder and avoiding operations in the high-dimensional space. They predict the evolution, even in time-extrapolation scenarios, of space-dependent fields without relying on predetermined grids, thus enabling weight-sharing across query-points. Lightweight and easy-to-train, Latent Dynamics Networks demonstrate superior accuracy (normalized error 5 times smaller) in highly-nonlinear problems with significantly fewer trainable parameters (more than 10 times fewer) compared to state-of-the-art methods.

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Domain decomposition for image classification problems

Janine Weber

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In many modern computer application problems, the classification of image data plays an important role. Among many different supervised machine learning models, convolutional neural networks (CNNs) and linear discriminant analysis (LDA) as well as sophisticated variants thereof are popular techniques. Divide-and-conquer algorithms in combination with machine learning methods have been proven to be an efficient approach for image classification problems yielding both, higher accuracy and good parallelization properties. In this talk, two different decomposed CNN models are experimentally compared for different image classification problems. Both models are loosely inspired by domain decomposition methods and in addition, combined with a transfer learning strategy. The resulting models show improved classification accuracies compared to the corresponding, composed global CNN model without transfer learning and besides, also help to speed up the training process. Moreover, a novel decomposed LDA strategy is discussed which also relies on a localization approach and which is combined with a small neural network model. In comparison with a global LDA applied to the entire input data, the presented decomposed LDA approach shows increased classification accuracies for the considered test problems.

This presentation is based on joint work with Axel Klawonn and Martin Lanser, University of Cologne, Germany.

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MS29 – Next-generation numerical methods for computational life sciences

Organizers: Francesca Bonizzoni, Mattia Corti, Ivan Fumagalli, Stefano Pagani

Advances in modeling and simulation of complex biological systems are unlocking new frontiers in healthcare, improving our understanding of physiological and pathological mechanisms, guiding drug design, and optimizing therapeutic interventions. The numerical solution of multiphysics problems arising in life sciences is particularly challenging due to the interplay of diverse spatial and temporal scales, the interaction of multiple physical laws, and the heterogeneous nature of biological tissues. Addressing these challenges requires next-generation numerical schemes that combine accuracy, efficiency, and robustness. This minisymposium focuses on cutting-edge numerical methods designed for applications in life sciences. Topics of interest include, but are not limited to: multiscale and multiphysics modeling, structure-preserving and robust numerical discretizations, efficient solvers and high-performance computing for coupled differential models, flexible mesh generation and adaptation strategies, high-order and polytopal methods, eXtended Finite Element Methods, adaptive FEM, CUTFEM, and data-driven methodologies. Special attention will be given to the challenges posed by complex geometries, non-conforming interfaces, and heterogeneous material properties. The discussion will benefit from contributions in applications such as soft tissue mechanics, molecular transport, fluid dynamics, and fluid-structure interaction in biological systems. By bringing together experts in numerical analysis, scientific computing, and computational biology, this minisymposium aims to foster collaboration and advance the development of innovative numerical techniques for real-world challenges.

List of Speakers

- D. Grappein (Politecnico di Torino) – *Optimization-based DD strategies for 3D-1D coupled problems: some applications in biological simulations*.
- C. B. Leimer Saglio (Politecnico di Milano) – *A p-adaptive high-order polytopal method for neuronal electrophysiology*.
- A. Ruggeri (Università degli Studi di Pavia) – *Impact of patient-specific parameters on biomechanical analysis during TEVAR follow-up*.
- G. Ziarelli (Università degli studi di Milano) – *Learning cardiac activation and repolarization times with operator learning*.

Optimization-based DD strategies for 3D-1D coupled problems: some applications in biological simulations

Denise Grappein

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An optimization-based domain decomposition approach for 3D-1D coupled problems is proposed as a strategy to reduce the computational burden in the simulation of biological processes that involve interactions between thin tubular components and surrounding living tissue. These components may include thin vessels in a capillary network or slender parts of a medical implant. In particular, we are interested in fluid, chemical, or heat exchanges occurring between the tubular components and the surrounding environment.

Under proper assumptions on the regularity of the solution, the problem of interest is reduced to a well-posed 3D-1D coupled problem, where the tubular inclusions are identified with their centerline. This is a common procedure when tackling this class of problems, that avoids the definition of a computational mesh inside the very thin vessels. Following a domain decomposition paradigm, two auxiliary variables are introduced at the interface, in order to decouple the problems in the tissue and in the tubular components. The problem is then solved in a PDE-constrained optimization framework, in which a proper cost functional, mimicking the error committed in the approximation of the interface conditions, is minimized subject to the 3D-1D set of equations. This results in a robust and flexible approach, especially suited for handling intricate capillary networks or complex medical implants. The use of the auxiliary variables ensures that the interface quantities are directly available without the need for post-processing.

In this talk, two examples of applications are proposed. An optimization based 3D-1D coupling for discontinuous solutions is applied to model fluid and oxygen exchanges between a growing capillary network and the surrounding tissue. The discontinuity at the interface allows for the modeling of the behavior of the capillary walls, which act as a semipermeable membrane. The dispersion of a growth factor driving capillary growth is also considered, whereas a discrete tip-tracking strategy, along with branching and anastomosis rules, is employed to simulate realistic vessel growth and network development. A continuous optimization-based 3D-1D coupling strategy is instead proposed for the simulation of the heating of tissues surrounding a metallic cranial grid subject to a magnetic field. The numerical results are compared to the data from experimental measurements, showing successful alignment.

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A p-adaptive high-order polytopal method for neuronal electrophysiology

Caterina Beatrice Leimer Saglio

Politecnico di Milano

Traveling wave phenomena, such as the propagation of electrical impulses in anisotropic tissues, are fundamental to many biological processes. In brain electrophysiology, these dynamics appear as sharp, fast-moving wavefronts of transmembrane potential, driven by ionic exchanges across cell membranes. Numerical simulation of such phenomena is key to understanding physiological and pathological brain activity, including the onset and spread of epileptic seizures. Accurately modeling these dynamics requires resolving multiple spatial and temporal scales within complex, anisotropic domains. High-order numerical methods like the discontinuous Galerkin (DG) method on polygonal and polyhedral grids (PolyDG) are well suited, as they can flexibly handle intricate brain geometries. However, uniform high-order discretizations are computationally prohibitive, especially for long-term simulations. This motivates the use of adaptive strategies that focus computational resources where needed.

In this work, we develop a novel p-adaptive PolyDG method for efficient and accurate simulation of brain electrophysiology. The method dynamically adjusts the local polynomial degree to capture steep wavefronts while reducing cost in quiescent regions. Central to our approach are a posteriori error indicators that detect wave-like features and guide automatic p-refinement and coarsening. A k-means clustering algorithm enhances efficiency by identifying candidate mesh elements for adaptivity. The underlying model couples the monodomain equation with the Barreto–Cressman ionic model, capturing ion-channel dynamics at the cellular scale. We validate the method through benchmark tests and simulations of epileptic seizure propagation in a realistic sagittal brainstem section. Results show that the adaptive strategy maintains high-order accuracy while significantly reducing computational cost, making it a powerful tool for scalable, high-fidelity brain simulations.

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Impact of patient-specific parameters on biomechanical analysis during TEVAR follow-up

Alessandro Ruggeri

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Cardiac and vascular diseases are the leading causes of death worldwide. Many cases of thoracic aortic disease, including aneurysms and dissections, carry a significant risk of morbidity and mortality. The use of endovascular devices has greatly enhanced the treatment of vascular diseases; however, many aspects are still poorly understood, and clinical studies often require a significant amount of time. Computational fluid dynamics (CFD) simulations offer a valuable tool for investigating these unknown aspects. For example, they can be used to study hemodynamics in patients' digital twins. There are several types of inlets and outlets boundary conditions (BCs) that can be applied to different problems, and various methods for estimating them can be found in the literature. However, selecting the appropriate boundary conditions is not straightforward. Nevertheless, using patient-specific clinical data to inform a simulation by setting the right boundary conditions can enhance our understanding of pathological progressions and the effects of specific treatments. In this discussion, we will explore various types of boundary conditions (BCs) and methods, along with their respective advantages and disadvantages. Additionally, we will illustrate how to enhance a simulation by selecting and tuning the appropriate boundary conditions based on the patient's available clinical data. A comprehensive methodological workflow will be presented, using a real clinical case scenario that involves the digital twins of patients who underwent thoracic endovascular aortic repair.

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Learning cardiac activation and repolarization times with operator learning

Giovanni Ziarelli

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Accurate modeling of cardiac electrophysiology through partial and ordinary differential equations (PDEs or ODEs) is crucial for early diagnosis and the development of clinical decision support systems. However, large-scale simulations, particularly on high-resolution meshes and realistic geometries, remain computationally intensive using traditional finite element methods (FEM). In this work, we present a systematic and comparative study on the application of two emerging operator learning techniques—Fourier Neural Operators (FNO) and Kernel Operator Learning (KOL)—to approximate the complex mapping from cardiac activation regions to activation and repolarization times. We evaluate these architectures on synthetic 2D and 3D domains as well as a physiologically realistic left ventricle geometry in terms of generalization error, training and inference times, and memory efficiency. Our results demonstrate that both FNO and KOL deliver significant computational speedups over FEM-based solvers while maintaining accuracy, and exhibit strong robustness to hyperparameter variations. These findings highlight the potential of operator learning to enable real-time, high-resolution simulations in cardiac modeling, paving the way for their integration into time-sensitive clinical workflows and large-scale personalized medicine applications.

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MS30 – Methods of reflection in the context of domain decomposition with applications

Organizers: Martin J. Gander, Laurence Halpern

This minisymposium gives an up to date overview of method of reflection techniques in the context of domain decomposition methods and recent applications, in particular wave propagation problems. The method of reflections goes back to Smoluchowski in 1911 who formulated it in rigorous mathematical terms for a Stokes problem, based on earlier work by Lorentz (particles in a liquid 1907), Lamb (Laplace problem 1906) and even Murphy (electrically charged particles 1833). The method of reflections was first an analytical method to obtain series expansions for scattering on multiple objects, by taking into account interactions one after the other, similar to the alternating Schwarz method, and became later only a numerical parallel solver technique, with a first parallel method of reflections introduced by Golusin (Laplace problem 1934). A convergence analysis using Schwarz techniques can be found in Luke (1989). For a complete historical review and references, see [1], and for a general formulation in terms of Schwarz methods, classical stationary iterations, scalability and preconditioning, see [2].

Over the last decade the method of reflections has become of interest in the domain decomposition community because domain decomposition techniques have become the best iterative solvers for the hard class of time harmonic wave propagation problems. There are however subtle differences between methods of reflections and domain decomposition methods, and the understanding and analysis of methods of reflections is trailing the rapid development of domain decomposition methods for such hard problems. The purpose of our minisymposium is to bring people together who work directly on the method of reflections, but also specialists in wave propagation and associated parallel solvers, with specific applications like elastodynamics, geophysics and optimal control, which could benefit from such new solver techniques.

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List of Speakers

- Y. Boubendir (NJIT) – *High-frequency multiple scattering equation methods with improved convergence properties.*
- L. Fesefeldt (Hamburg University of Technology) – *Acceleration of Newton's method in structural mechanics with p-FEM initializations.*
- M. J. Gander (University of Geneva) – *On the method of reflections and its relation to Schwarz methods.*
- J. Salomon (CNRS, INRIA, LJLL, EPC ANGE) – *The method of reflections in the Neumann case.*

High-frequency multiple scattering equation methods with improved convergence properties

Yassine Boubendir

NJIT

This talk is concerned with the acceleration of the multiple scattering iterative procedure based on integral equations and an adapted asymptotic expansion of the Helmholtz solution in the case of convex obstacles. We present an effective Krylov subspace method that significantly accelerates the convergence of the obtained Neumann series. This preconditioner is well adapted to the high-frequency aspect of the scattering problem as it retains the phase information associated with the iterates and delivers highly accurate solutions in a small number of iterations. A new kind of dynamical preconditioner based upon Kirchhoff approximations and the stationary phase method is explained. In addition, we present preliminary results for accurate approximation of the remaining infinite tail in the Neumann series formulation.

Joint work with F. Ecevit.

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Acceleration of Newton's method in structural mechanics with p-FEM initializations

Lina Fesefeldt

Hamburg University of Technology

We consider displacement problems in structural mechanics using a hyperelastic material model. Newton's method is applied to obtain a linearized problem. The convergence of Newton's method depends strongly on the choice of the initial guess. In case of divergence, load steps can be used to stabilize the method: The force acting on the body is applied incrementally, and each solution to the subproblems serves as a new starting vector for the next load step. While this method is intuitive and well established, it is also computationally and memory intensive. Using a benchmark problem from computational mechanics, we show how the computational complexity can be reduced if the p-version of the FEM is used. We exploit the hierarchical structure of the high-order FE problem to efficiently construct initial vectors for Newton's method.

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On the method of reflections and its relation to Schwarz methods

Martin J. Gander

University of Geneva

The method of reflection was invented to obtain approximate solutions of the motion of more than one particle in a given environment, provided that one can represent the solution for one particle rather easily. This motivation is quite similar to the motivation of the Schwarz domain decomposition method, which was invented to prove existence and uniqueness of solutions of the Laplace equation on complicated domains, which are composed of simpler ones, for which existence and uniqueness of solutions was known. Like for Schwarz methods, there is also an alternating and a parallel method of reflections, but interestingly, the parallel method is not always convergent. I will explain in my presentation the historical development of these methods of reflections, give several precise mathematical formulations, an equivalence result with the alternating Schwarz method for two particles, and also an algebraic formulation of these methods for solving Helmholtz and elasticity problems.

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The method of reflections in the Neumann case

Julien Salomon

Sorbonne Université, Université Paris Cité, CNRS, INRIA, Laboratoire Jacques-Louis Lions, LJLL, EPC ANGE

The method of reflections is an algorithm that approximates the solution of particle interaction problems by iteratively solving problems whose complexity is that of a one-particle problem. A general way to prove the convergence of this method consists in interpreting it as an alternating (or cyclic) projections algorithm. In such a framework, the proof can be obtained by showing that the projections are orthogonal. Such an approach has been used in a previous paper to prove the convergence in the cases where the boundary conditions associated with the particles are of Dirichlet type and of fourth type. In this talk, we prove the convergence of the method of reflections in the case of Neumann boundary conditions. This proof requires a specific construction since the projectors associated with these conditions are in general not orthogonal. However, we show that they are orthogonal when restricted to certain spaces in which the iterative update of the method takes place.

Joint work with P.-H. Cocquet and G. Legendre.

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MSHI – High-performance computing and industrial applications

Organizers: Paola F. Antonietti, Michele Botti, Gabriele Ciaramella

Technologies based on high-performance computing (HPC) such as numerical simulations, data analytics, and artificial intelligence (AI) are nowadays fundamental tools for industrial processes, predictive control, and data-driven decisions. The exponential growth in computing power of supercomputers requires numerical algorithms supporting parallel implementations. Domain decomposition solvers have shown to be very efficient in exploiting HPC capabilities by solving coarse problems in a highly scalable way. In several applied contexts, such as medicine, smart cities, and environmental monitoring, HPC is also mandatory to store and process Big Data that are aggregated and analyzed using AI techniques.

This mini-symposium focuses on the recent developments among the HPC, Big Data, and AI technologies towards the exascale era. An overview of currently available methods that fit with this concept will be presented together with examples of their actual use in industrial applications by the leading HPC technology suppliers and stakeholders.

List of Speakers

- M. Busetto (ABB Corporate Research) – *A two-step method coupling magneto-static and magneto-quasistatic formulations.*
- P. Fischer (University of Illinois, Urbana-Champaign) – *High-order Schwarz methods for incompressible flow applications.*
- C. Janna (M3E S.r.l.) – *Chronos: a GPU-accelerated general purpose AMG solver for industrial applications.*
- E. Manuzzi (MathWorks) – *Machine learning driven mesh refinement and agglomeration with MATLAB.*
- A. Mauri (Micron Technology and Development) – *Nanoscale memory simulation: analogies with biophysical systems and emerging frontiers.*
- P. Panarese (MathWorks) – *Scientific machine learning and high-performance computing in MATLAB.*
- C. Scrofani (Eni S.p.A.) – *Integration of a parallel geomechanical solver into a proprietary basin modelling software suite.*
- N. Tardieu (EDF R&D) – *Legacy codes in industry in the age of HPC.*

A two-step method coupling magneto-static and magneto-quasistatic formulations

Martina Busetto

ABB Corporate Research, Switzerland

When simulating electric arcs in switching devices, electromagnetic fields need to be computed in a domain involving moving contacts. In the moving parts of the domain the problem formulation can be complicated due to remeshing and field interpolation. Consequently, a first simplified simulation approach consists in neglecting the transient electromagnetic effects in the whole computational domain. However, this approach does not allow to properly capture the sizeable effects that eddy currents may have on the Lorentz forces acting on the plasma arc. Consequently, a more refined simulation approach would need to include transient effects. The effects of eddy currents are expected to be larger in the non-moving ferromagnetic parts due to the high magnetic permeability, much smaller in the moving copper contacts, and negligible in the plasma arc. Hence, the largest effects of eddy currents are expected in the non-moving metallic parts of the switching device. Therefore, in this work we introduce the mathematical formulation and the numerical validation of a method tailored to include eddy current effects only in a part of the domain which does not change over time. This results in a heterogeneous two-domain problem combining a magneto-quasistatic model in a subset of the computational domain with a magneto-static model in the remainder of the domain. We adopt a two-step approach in which the magneto-static fields computed in the first step are corrected for inductive phenomena in the second step. The primary variables of the problem are the electric scalar potential and the magnetic vector potential. We discuss boundary conditions and the limitations of the approach, and we show numerical results that validate the formulation.

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High-order Schwarz methods for incompressible flow applications

Paul Fischer

University of Illinois, Urbana-Champaign

We discuss the role of domain decomposition in the development of the open-source code, Nek5000/RS, which targets incompressible and low-Mach thermal-fluid simulations. The underlying discretization is the spectral element method (SEM) introduced in [4], which is a finite element method that realizes high-order convergence through the use of efficient tensor-product forms noted by [3]. In the 3D reference element, $[-1:1]^3$, the number of memory accesses for operator evaluation is only a small constant times the number of unknowns, $n = EN^3$, for E elements having local Lagrange polynomial bases of order N in each direction. The work for operator evaluation is also low, only $O(EN^4) = O(nN)$, and can be cast as cache-friendly BLAS3 dgemm operations [1]. In practice, approximation order $N = 7$ is most common.

For the incompressible (or low-Mach) Navier-Stokes equations, the stiffest substep in time advancement is the pressure Poisson solve, which enforces the divergence-free constraint. We have developed several Poisson preconditioners for Nek5000/RS. The most widely used is based on multilevel overlapping Schwarz in either an additive or multiplicative form. In the multiplicative form, we have recast the Schwarz component as part of a Chebyshev-accelerated smoother within p-multigrid. The overlapping subproblems are defined on "single-node" extensions of each spectral element into its six neighbors. As there are $(N+1)^3$ unknowns per element (here, subdomain) the overlapping problem requires determination of $(N+3)^3$ unknowns. In a regular domain, the local Poisson problem is separable and can be solved using fast tensor product solvers (e.g., [2]). For deformed elements, solving a nearby separable problem with these fast methods is suitable for preconditioning/smoothing.

The second application of domain decomposition in Nek5000/RS is to use overlapping Schwarz at the full Navier-Stokes level. In many industrial applications, it is cumbersome to have a single monolithic mesh cover the entire domain. The Schwarz overlapping method provides an efficient mechanism to support nonconforming meshes. One simply assigns two or more (overlapping) subdomains to separate sets of (private memory) processes and advances the Navier-Stokes equations in each of these. Boundary data for boundary subsets that lie in the complementary domains are found using interpolation of prior timestep information. Stable high-order temporal accuracy can be realized with a predictor corrector step. Moreover, this approach can be used to effectively implement multi-rate timestepping, which is a significant challenge in the context of incompressible flow solvers.

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Chronos: a GPU-accelerated general purpose AMG solver for industrial applications

Carlo Janna

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Algebraic Multigrid (AMG) is a very popular iterative method used in several applications. This wide diffusion is due to its effectiveness in solving linear systems arising from PDEs discretization. The key feature of AMG is its optimality, i.e., the ability to guarantee a convergence rate independent of the mesh size for different problems. This is obtained through a good interplay between the smoother and the interpolation. Unfortunately, for difficult problem, such as those arising in industrial applications, standard AMG techniques are not effective and more elaborated strategies and algorithms need to be developed. The implementation of novel AMG methods became even more difficult with the advent of manycore hardware such as GPU accelerators. While AMG application simply consists in matrix by vector products that have been already successfully ported on GPU by a number of authors, the set-up stage needs to be completely redesigned. In this work, we present the AMG preconditioner included in Chronos, tailored for industrial applications from both solid mechanics and CFD and entirely running on GPU. Thanks to some numerical experiments we will show its performance and scalability on real-world problems.

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Machine learning driven mesh refinement and agglomeration with MATLAB

Enrico Manuzzi

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Adaptive mesh refinement and agglomeration are critical for enabling efficient and scalable solvers in complex physical simulations, particularly within multigrid and domain decomposition frameworks. In this talk, we present Machine Learning (ML)-driven strategies for both refinement and agglomeration on general polygonal and polyhedral grids in 2D and 3D. Element-wise refinement is guided by Convolutional Neural Networks (CNNs), while Graph Neural Networks (GNNs) and clustering algorithms facilitate high-quality agglomeration, yielding effective coarse representations for numerical solvers. We evaluate these ML approaches using quality metrics, mesh complexity, computational cost, and solver performance, demonstrating robust adaptability to intricate geometries. MATLAB toolboxes, including the Deep Learning Toolbox, provide a powerful environment for the development, training, and application of these ML models within simulation workflows. MATLAB accelerates rapid prototyping, visualization, and integration of advanced ML-based mesh processing for both 2D and 3D finite element analyses. We will showcase how ML enhances mesh refinement and agglomeration, and discuss the advantages of MATLAB for integrating these innovative techniques into computational science and engineering applications.

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Nanoscale memory simulation: analogies with biophysical systems and emerging frontiers

Aurelio Mauri

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In the semiconductor industry, mathematical simulations based on both ordinary and partial differential equations have been extensively used in the past and continue to be used today, primarily for developing new electronic devices and interpreting electrical responses. However, in the present day, the current scaling architecture of the most advanced memory devices requires robust computational and mathematical efforts. On one hand, the extensive use of the third dimension for both CMOS and memory device scaling has transitioned the traditional 2D computational domain to a fully 3D framework. On the other hand, exploring new frontiers necessitates modeling new physical phenomena that have not been fully considered until recently, especially in the scaling of memory devices. In traditional electronic device simulation, a fundamental approach relies on the Drift-Diffusion model, which has been modified with several corrections to simulate the most advanced devices. We discuss several significant analogies with the Poisson-Nernst-Planck (PNP) approach used for modeling biophysical systems, such as ion dynamics in cellular membranes. The evolution of semiconductor memory is a broad field, and even with corrections, the Drift-Diffusion model cannot meet current demands, particularly when examining the transport of charge ions in chalcogenide materials used in phase-change memory. In these materials, electric resistivity changes between the amorphous and crystalline phases. Continuing with the analogy, the homogeneous charged mixture approach has been successfully applied to study the modification of metal alloys during memory usage. More recently, cation-based resistive memories have gained increasing interest in neuromorphic applications due to their ability to emulate the dynamic behavior of neurons and synapses. The physical modeling of these devices relies on the formation and dissolution of a conductive metal filament within an insulator layer encapsulated between two metal electrodes. To comprehensively model the shape evolution of such a filament in the presence of a strong electric field, while simultaneously describing the electric status of the memory cell, a mixed finite element method with mesh adaptation is successfully employed. The results of such an application are also presented.

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Scientific machine learning and high-performance computing in MATLAB

Paolo Panarese

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Recent advances in scientific machine learning (SciML) are reshaping computational science, enabling the efficient solution of complex physical problems. This talk introduces the extended Deep Learning framework in MATLAB, highlighting its support for automatic differentiation and custom training loops tailored to scientific workflows. Starting from a classical partial differential equation (PDE), we illustrate how to construct an adjacency matrix from the computational mesh and define a graph neural network (GNN) architecture. This approach turns out to achieve faster inference than traditional finite element method solvers while maintaining accuracy. We then provide a high-level overview of advanced SciML techniques for embedding domain knowledge and constraints into learning pipelines. This includes physics-informed neural networks (PINNs), which encode PDEs directly in custom loss functions, and architectures that enforce monotonicity, convexity, or Lipschitz continuity to ensure consistent behaviors in safety-critical applications. Finally, we briefly highlight High-Performance Computing (HPC) strategies in MATLAB that facilitate scalable simulations. Leveraging the combined memory of machine clusters, we demonstrate how to solve large linear systems arising from finer mesh discretization by using multigrid preconditioners and distributed arrays.

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Integration of a parallel geomechanical solver into a proprietary basin modelling software suite

Giovanni Scrofani

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Basin modelling (BM) is an Earth Sciences discipline dealing with the study of multiphysics processes governing sedimentary basins formation and evolution. It can provide valuable information contributing to reduce uncertainties in exploration for natural resources. Since 80s, Eni has developed and implemented its own proprietary basin modelling suite, e-simba®.,. An important component of BM is pressure and temperature (P&T) computation of the fluid occupying rock pores, during basin evolution. Eni's P&T solver TemPrA is based on a Finite Element (FE) discretization and is implemented in a parallel environment. This framework fully benefits from the computational power of the large cluster Eni's Green Data Center. The traditional approach in BM is based on the Terzaghi formulation, where the maximum principal stress is given by the vertical load due to the weight of overlying saturated terrains. One of the latest improvements at basin scale in e-simba® is the introduction of a more accurate geomechanical modeling tool which takes into account the lateral components of the stress. This is achieved by coupling TemPrA solver with ATLAS, a 3D FE software tailored for geomechanical applications, such as simulation of land subsidence, fault activation, and other relevant geomechanical processes occurring in the subsurface. ATLAS implements advanced algorithms for the solution of numerical problems arising from the discretization of geological domains. The software library includes several constitutive laws for materials and different types of elements, including interface elements to simulate faults and discontinuities. ATLAS is natively designed for parallel supercomputers and takes advantage of GPU-CPU hybrid architectures, easily allowing the simulation of extreme-size domains with several hundreds of millions of elements. The ATLAS library has been successfully integrated into TemPrA to calculate the effective stress generated within the layers of a sedimentary basin during its evolution. The coupling of the two codes involves 3 steps: a first preprocessing phase, where the input parameters for ATLAS, namely the geometry of the model, the initial stress state, the applied loads and the boundary conditions are prepared; a second phase where the new tensional state is computed by ATLAS; a final phase which provides the results from the external library and updates the variables in TemPrA. This integration provides more accurate results, as the effective stress is evaluated as a tensor, instead of the scalar approximation based on the assumptions of the Terzaghi law.

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Legacy codes in industry in the age of HPC

Nicolas Tardieu

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Computational software is a cornerstone in all sectors of industry. It is used throughout the production cycle, from design to manufacturing to failure analysis. In the context of solid mechanics, legacy codes, i.e. software that was originally designed and developed in the last century and has undergone several revisions in order to remain usable and performant, are very common. As an example, code_aster, a legacy code developed since 1989 by the company Electricité de France (EDF), one of the largest utilities in the world, is able to run multi-physics, multi-finite elements (mixing of continuum mechanics, beams, and shells) including multi-point constraints in a fully parallel workflow, from mesh reading to result writing. In this talk, we will highlight some successes, such as solving coupled thermo-hydro-mechanical problems exceeding 1 billion degrees of freedom on 2000+ cores, and also discuss some persistent challenges.

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Contributed Talks

List of Contributed Talks

- M. Babamehdi (Bergische Universität Wuppertal/Forschungszentrum Jülich) – *Subspace correction methods in nonlinear problem*.
- L. Badea (Institute of Mathematics of the Romanian Academy) – *Convergence analysis of classical and multilevel Uzawa algorithms for general saddle point problems*.
- J. Galvis (Universidad Nacional de Colombia) – *On condition numbers of symmetric and nonsymmetric domain decomposition methods*.
- M. Hanek (Institute of Mathematics of Czech Academy of Sciences) – *Speeding up an unsteady flow simulation by the adaptive BDDC and Krylov subspace recycling*.
- I. Igreja (Federal University of Juiz de Fora) – *Stabilized mixed hybrid finite element methods for the Stokes-Darcy coupled problem*.
- P. Matalon (CMAP, Ecole polytechnique) – *Weighted GMRES and prescribed convergence curves*.
- E.-J. Park (Yonsei University) – *Adaptive multi-level Newton algorithm for a class of nonlinear problems*.
- A. Saponaro (Politecnico di Torino) – *FETI method for the solution of structural dynamics problems involving localized nonlinearities*.
- T. Seibel (CEM at TU Darmstadt) – *On a hybridized domain decomposition formulation*.
- R. Shen (School of Mathematics and Computing Science, Guilin University of Electronic Technology) – *An inverse averaging finite element method for solving PNP/SMPNP equations in ion channel simulations*.

Subspace correction methods in nonlinear problem

Mehdi Babamehdi

Bergische Universität Wuppertal/Forschungszentrum Jülich

Subspace correction methods can be powerful iterative methods. Although it is not a new idea to use the methods in solving nonlinear problems, it is really difficult to predict the efficiency of the method in different nonlinear problems [1], since the characteristics of the nonlinear equations can significantly change from one problem to another. In this study we look at the nonlinear multigrid (FAS) [2], as multilevel successive subspace correction method for some nonlinear PDEs. One of the important components of the multigrid method is a proper smoother operator which should work "well" with the other component of multigrid, i.e. the coarse grid correction operator. As a smoother for FAS we used the nonlinear additive Schwarz method (ASM), as parallel subspace correction [3]. We investigated the smoothing condition of the nonlinear ASM method as a smoother in FAS and then used the algorithm to solve some nonlinear PDEs in GPU implemented in a matrix-free form.

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Convergence analysis of classical and multilevel Uzawa algorithms for general saddle point problems

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We analyze the convergence of classical and multilevel variants of the Uzawa algorithm for general saddle point problems. The multilevel variant has similar iterations to the classical one, but the solution of the first equation is only approximated by inner iterations. For both classical and multilevel methods, explicit formulas for the convergence conditions and convergence rates are given. Using these results, we prove that when the number of inner iterations of the multilevel Uzawa method tends to infinity, its convergence condition and convergence rate coincide with those of the classical Uzawa method. Furthermore, by writing the convergence rate of the multilevel method in terms of the inner iterations, we show that this either has a minimum point or is an increasing function. We can conclude that the multilevel method with very few inner iterations converges better than the classical one. Numerical experiments carried out for the driven-cavity Stokes problem are in a very good agreement with the theoretical results.

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On condition numbers of symmetric and nonsymmetric domain decomposition methods

Juan Galvis

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Using oblique projections and angles between subspaces we write condition number estimates for abstract nonsymmetric domain decomposition methods. In particular, we consider a restricted additive method for the Poisson equation and write a bound for the condition number of the preconditioned operator. We also obtain the non-negativity of the preconditioned operator. Condition number estimates are not enough for the convergence of iterative methods such as GMRES but these bounds may lead to further understanding of nonsymmetric domain decomposition methods. Based on the paper [1].

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Speeding up an unsteady flow simulation by the adaptive BDDC and Krylov subspace recycling

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We study the acceleration of an iterative method for sequences of linear systems for unsteady incompressible flow problems governed by the Navier-Stokes equations. We compare subspace recycling within the deflated preconditioned conjugate gradient method and adaptive selection of coarse space in BDDC and their combination. We solve the Navier-Stokes equations by the pressure-correction scheme in connection with the finite element method. This approach leads to sequences of linear systems over the time steps. Our particular interest is the Poisson problem of pressure. Results for the problem of flow around the sphere for Reynolds numbers 100 (steady flow) and 300 (unsteady flow) are presented. We demonstrate that by using these approaches, we are able to save one half of the computational time.

Joint work with J. Papež and J. Šístek.

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Stabilized mixed hybrid finite element methods for the Stokes-Darcy coupled problem

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This work presents stabilized hybrid mixed finite element formulations for simulating the interaction between free-flow and porous media governed by the Stokes and Darcy equations in mixed velocity-pressure form. In the Stokes region, the Lagrange multiplier is consistently defined as a vector field associated with the velocity, ensuring the continuity of momentum across the interface. In the Darcy domain, different choices of multipliers are investigated. The first approach uses the full Darcy velocity as the multiplier; the second uses only the normal component of the Darcy velocity; and the third defines the multiplier as the Darcy pressure. These distinct strategies provide flexibility in satisfying the interface conditions and significantly impact the structure, conditioning, and size of the resulting global system. The global system is assembled by eliminating the local degrees of freedom associated with velocity and pressure through static condensation at the element level. This results in a reduced system involving only the multipliers, improving computational efficiency and enabling a modular formulation. The interface conditions between Stokes and Darcy domains are naturally enforced through the proper choice of multipliers, without requiring penalty parameters or artificial constraints. The stability of the formulations is supported by theoretical results available in the literature. A series of numerical experiments in two and three dimensions are conducted to assess their performance in both homogeneous and heterogeneous porous media. The results confirm that the different formulations achieve accurate approximations and optimal convergence rates for velocity, while the pressure convergence may depend on the choice of multiplier in the Darcy region. Moreover, the comparison between the formulations highlights how the choice of interface variables can be tailored to balance complexity, performance, and robustness. Finally, this study opens the way for exploring new combinations of multipliers that may further improve the stability and convergence behavior of hybrid methods for coupled Stokes–Darcy problems in complex domains.

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Weighted GMRES and prescribed convergence curves

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[1] states that from any prescribed convergence curve, one can build a linear system for which GMRES realizes that convergence curve. We build upon this idea to derive novel results about weighted GMRES. We prove that for any linear system and any prescribed convergence curve, there exists a weight matrix M for which weighted GMRES (*i.e.* GMRES in the inner product induced by M) realizes that convergence curve, and we characterize the form of M. Additionally, we exhibit a necessary and sufficient condition on M for the simultaneous prescription of two convergence curves, one realized by GMRES in the Euclidean inner product, and the other in the inner product induced by M. Finally, these results are applied to infer some properties of preconditioned GMRES.

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Adaptive multi-level Newton algorithm for a class of nonlinear problems

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Combining adaptive mesh refinement with Newton's method provides a powerful approach for efficiently solving nonlinear PDEs, particularly in the presence of localized singularities and boundary layers. The synergy between these techniques addresses two key challenges: (1) capturing fine-scale features through mesh adaptation, and (2) accelerating nonlinear convergence using Newton-type iterations.

In this talk, we present a unified a posteriori error estimation framework for multilevel Newton algorithms within the Brezzi-Rappaz-Raviart (BRR) theoretical setting. By integrating two-grid initialization with adaptive refinement, we achieve quadratic convergence across mesh levels—offering significant advantages for solving nonlinear problems such as the Navier-Stokes equations (NSE) and semilinear elliptic systems. To further enhance efficiency, we also explore adaptive finite element methods (AFEM) that incorporate smoothing iterations in the nonlinear context.

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FETI method for the solution of structural dynamics problems involving localized nonlinearities

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Nowadays, given the availability of High Performance Computing environments and given the industrial need of performing analyses on large mechanical systems, domain decomposition methods represent a suitable and well established class of techniques for the solution of large scale problems. One of the engineering fields, where the state-of-the-art approaches limit the size of the studied finite element (FE) models, is the structural dynamics addressing the study of nonlinear problems involving localized contact nonlinearities.

To save computational time, nonlinear dynamic analyses are performed in the frequency domain using the harmonic balance method (HBM), linearizing the equilibrium equations of motion using the Newton-Raphson technique. Focusing on each iteration of the Newton-Raphson, the linear equation to be solved has the form $[D(w)+J(x)]*(delta_x) = (-D(w)*x+f-fnl(x))$ where w is a positive scalar, $D(w)$ is a complex non-Hermitian matrix (quadratically depending on w) and $J(x)$ is the Jacobian of the nonlinear term $fnl(x)$. The Jacobian $J(x)$ is computed in real algebra using the alternating frequency/time (AFT) procedure, arising the need of solving the previous equation in real algebra. To reduce the size of the previous linear system, state-of-the-art approaches make use of reduced order models (ROMs) which must satisfy some physical assumptions and need a reduction/expansion process of the solution (not always feasible being high demanding from the computational point of view), consequently limiting the size of the studied FE models.

The proposed approach is based on the finite element tearing and interconnecting (FETI) technique (consisting in the decomposition of FE model into multiple domains), applied to the solution of the linearized equilibrium equation solved in the frequency domain. Due to the nature of the problem, the application of the FETI methods leads to the solution of a non-symmetric indefinite linear system in turn solved with the GMRES, which, given the properties of the involved matrices, requires the introduction of a suitable preconditioner introduced by deflation. It is also good to notice that, given the localized nature of the nonlinearities, this approach defines a small number of domains “equipped” with the nonlinear term $J(x)$ which should be considered in the decomposition of the FE model.

The adoption of this approach removes all the assumptions that must be satisfied when ROMs are applied and removes the need of a reduction and expansion process requested by the adoption of ROMs, additionally simplifying the solution process from the user point of view.

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On a hybridized domain decomposition formulation

Timon Seibel

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In this talk, we introduce a mixed variational formulation for domain decomposition on non-overlapping patches, which is naturally well-posed on each patch. The formulation comes with a hybrid variable μ in $L^2(\Gamma)$ on the skeleton Γ. Inspired by Hybridized Discontinuous Galerkin (HDG) methods, μ acts as the only primal variable. This is in contrast to FETI-DP, where the variables on the patches are partially dual and primal, while all variables on the patches in our approach are dual variables. The resulting Schur complement system for μ on Γ turns out to be self-adjoint. We apply this approach — hereafter referred to as Hybridized Domain Decomposition (HDD) — to the generalized Poisson equation "$-\mathrm{div}(\kappa \nabla u) = f$" in two dimensions. For each patch, we consider both solution components u and q, where $q = \kappa \nabla u$.

The main contributions of this work are threefold: First, we establish the well-posedness of the HDD formulation and, in particular, we prove that the mixed problem is well-posed on each individual patch. Second, we introduce higher-order finite element discretizations, in which the solution components u and q are approximated by piecewise discontinuous polynomials and $H(\mathrm{div})$-conforming vector fields, respectively, while μ is discretized using discontinuous piecewise polynomials on the skeleton. We provide a rigorous a priori error analysis with respect to mesh refinement, demonstrating that the discretization error is optimal in the sense that it matches the best approximation error. Third, we prove that the bilinear form of the Schur complement problem for μ on the skeleton is symmetric and $L^2(\Gamma)$-elliptic, ensuring well-posedness and enabling efficient solution by the preconditioned CG method.

Our numerical results show high-order convergence in the mesh width for smooth domains with curved meshes. For an L-shaped domain with curved boundaries we observe convergence rates that are restricted by the solution's singularity.

In summary, the HDD approach provides a robust and efficient framework for domain decomposition, combining the advantages of HDG methods with the flexibility of patch-based decomposition and a well-posed mixed formulation. The method is particularly well-suited for problems with complex geometries and allows for efficient parallelization via the interface Schur complement, while our a priori error estimates confirm optimal convergence rates in agreement with the best approximation error.

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An inverse averaging finite element method for solving PNP/SMPNP equations in ion channel simulations

Ruigang Shen

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In this talk, an inverse averaging finite element method (IAFEM) is introduced for solving the PNP/SMPNP equations. By introducing a generalized Slotboom transform, each of the size-modified NP equation is transformed into a self-adjoint equation with exponentially behaved coefficient. However, in traditional numerical schemes, the Slotboom variable can lead to an ill-conditioned stiffness matrix, which is not conducive to the scalability and application of solvers. By employing our recently developed inverse averaging technique to deal with the exponential coefficients of the reformulated formulations, featured with advantages of numerical stability and flux conservation especially in strong nonlinear and convection-dominated cases. It is more concise and easier to be numerically implemented. The new inverse averaging technique provides a new reference for the development of numerical solvers for biomolecular systems.

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Posters

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- F. Brunelli (Laboratoire Jacques-Louis Lions, Sorbonne University) – *A scalable domain decomposition method for saddle point problems with GenEO coarse spaces.*
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- K.-H. Kim (Division of National Supercomputing, Korea Institute of Science and Technology Information KISTI) – *A domain-decomposed parallel solver for block tridiagonal matrix systems on distributed memory architectures.*
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- N. Kumar (Politecnico di Milano) – *Domain decomposition based nodal integration method for Helmholtz equation.*
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- Y. Yang (University of Macau) – *Numerical studies of the Brugada syndrome with a parallel bidomain model.*
- B. Zheng (LSEC, ICMSEC, AMSS, CAS) – *One-level integrated additive domain decomposition solver for LOD coarse problem for Helmholtz problem with high wave number.*

Optimization of OSWR convergence taking into account Robin initial data

Arthur Arnoult

Université Sorbonne Paris Nord

In this poster, we present a new analysis of Optimized Schwarz Waveform Relaxation (OSWR) algorithm for the 1D heat equation, i.e. with Robin-type transmission conditions applied over the whole time window. Our approach is based on a time discretization using a numerical scheme, followed by a global analysis of the error vector over all time steps. This methodology leads to a new analysis of the algorithm's convergence, and to new results on its behavior.

On the other hand, the Robin data of the algorithm is usually initialized by applying the Robin operator to the initial condition, which favors rapid convergence. However, the usual choice of Robin parameter does not take this particular initialization into account, but is obtained by a Fourier time transform technique in which all temporal frequencies are treated identically. In practice, however, we observe that the optimal parameter is highly dependent on the initialization of the algorithm.

We therefore propose a new methodology, based on the discrete-time analysis and a Taylor expansion of the Robin condition around its initial value. Numerical results will illustrate the optimality of the parameter obtained, leading to a significant gain in terms of convergence speed of the OSWR algorithm.

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A scalable domain decomposition method for saddle point problems with GenEO coarse spaces

Filippo Brunelli

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We present an adaptive domain decomposition (DD) preconditioning technique for the solution of saddle point problems with a 2x2 blocks structure. This work utilises the GenEO theory for symmetric positive definite (SPD) problems (Spillane et al., 2014) to tackle the solution of saddle point linear systems with iterative methods. The latters are preferred over direct methods due to the typically large size of these problems. If we assume all matrices to be sparse and the diagonal blocks to be also spectrally equivalent to sums of symmetric positive semidefinite matrices, then the derived preconditioners are fully algebraic and scalable. These assumptions are also at the basis of GenEO method. We will present a preconditioner for the primal-primal block and one for the Schur complement matrix, both based on the DD paradigm. Then, these ingredients are used in the context of preconditioning techniques for the 2x2 block saddle point matrix. Numerical tests are performed on two classical systems of partial differential equations (PDEs) discretized with finite elements, for which the above assumptions are easy to verify. The results emphasize preconditioner's robustness with respect to the specific domain's partitioning.

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DDM in the derived-vector space DDM-DVS and its applicability to nonsymmetric matrices

Alicia de la Mora Cebada

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This poster presents a review of the Domain Decomposition Method in Derived Vector Space (DDM-DVS), originally proposed by [1]. DDM-DVS is a domain decomposition method that employs non-overlapping subdomains and non-overlapping nodes, distinguishing it from standard Iterative Substructuring Methods, in which boundary nodes between subdomains typically overlap. The discretized problem is solved in an extended vector space called the Derived Vector Space (DVS), where the system matrix becomes block-diagonal. This structure offers significant advantages for implementing algorithms in high-performance parallel computing, as it allows each processor to independently solve a single subdomain. The main contributions include the development of the DVS-Schur and DVS-BDDC methods for non-symmetric matrices. These algorithms operate on three levels: the first globally solves for dual nodes, the second globally solves primal nodes, and the third locally solves interior nodes within each subdomain. It is also shown that the Schur complement of the system matrix preserves a block-diagonal. The equivalence of the discretized problem is demonstrated both in its standard formulation and within the DVS framework. The superlinear performance of these algorithms is confirmed and analized through their algorithmic complexity, identifying operational cases based on the ratio of dual to interior nodes. Numerical examples using the general second-order differential operator are included. Systems with up to 100 million degrees of freedom have been succesfully solved using as many as 441 processors.

Joint work with E. Rubio.

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Practical impact assessments by domain decomposition for enhanced modelling in Lake IJssel and Lake Marken - illustration for Marker Wadden

Menno Genseberger

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Currently, several practical developments related to different societal aspects are being investigated for Lake IJssel and Lake Marken in the Netherlands. To facilitate this with integral impact assessments, there is a need of hydrodynamic and water quality models that have enough quality and resolution and are fast enough on modern hardware. A typical and very actual example is the extension of Marker Wadden, a collection of archipelago type of islands, in the east of Lake Marken. This poster illustrates how for this example, a silt model of Lake Marken [1] is enhanced by means of domain decomposition. Hourly coupled flow wave computations are performed in the model. Flow computations are with a shallow water solver from Delft3D(4) [2], wave computations are with SWAN [3]. The wave part uses online nesting for the required local resolution and fast computing on modern hardware with many cores within shared memory. For the flow part, a domain decomposition approach in Delft3D(4) is applied for local grid refinement around the islands of Marker Wadden. Additionally, the subdomain with the refined grid is further decomposed in stripwise subdomains to enable fast computations on the solver level in shared memory.

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A domain-decomposed parallel solver for block tridiagonal matrix systems on distributed memory architectures

Ki-Ha Kim

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Block tridiagonal matrix systems frequently arise in numerical solutions of multi-dimensional partial differential equations, particularly when applying high-order finite difference schemes or nonlinear implicit formulations. Solving these systems efficiently in large-scale, batch-processing scenarios is critical in modern scientific computing. In this work, we present a novel domain-decomposed parallel solver tailored for block tridiagonal matrix systems on distributed memory architectures. By extending the PaScaL_TDMA algorithm, originally developed for scalar tridiagonal systems, to handle matrix blocks, our approach leverages the Modified Block Thomas Algorithm and minimizes inter-process communication via a two-level reduction and back-substitution strategy. Matrix operations are performed using optimized BLAS and LAPACK routines, and our implementation demonstrates excellent strong scalability—achieving over 98% efficiency across thousands of cores. This solver design aligns naturally with the philosophy of domain decomposition methods and is well-suited to applications in large-scale simulations requiring high throughput and parallel performance. The results contribute to both the theoretical and practical development of decomposition strategies in scientific computing and high-performance environments.

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Coupling domains with Chorin-Temam projection in the OSWR method

Valentin Kraemer

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Solving a monolithic velocity-pressure system for the unsteady Navier-Stokes equations becomes challenging as the number of unknowns significantly increases. Decoupling velocity and pressure using projection methods, such as the Chorin-Temam projection method, is a popular approach in thermo-hydraulic codes including TRUST-TrioCFD code. This poster proposes a nearly embarrassingly parallel method for high-performance computing (HPC) by coupling the projection method with a non-overlapping Schwarz waveform relaxation method using Robin transmission conditions. Such coupling is studied in the context of a Finite Element Volume discretization, which is equivalent to a Finite Element method with Crouzeix-Raviart elements. The equivalence between the monodomain system and the multidomain formulation with Robin transmission conditions is demonstrated, and several data transmission strategies are presented in the poster. Robin boundary conditions in the domain decomposition perspective have been implemented and validated in both 2D and 3D for the Stokes problem within the TRUST code.

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Domain decomposition based nodal integration method for Helmholtz equation

Niteen Kumar

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The Helmholtz equation finds applications in various fields, including neutron transport, wave-field simulation, radiation modeling etc. However, obtaining accurate solutions remains a significant challenge. Numerical solutions of the Helmholtz equation often encounter issues related to singularities. To address this, we developed a domain decomposition-based Nodal Integral Method (NIM) for solving the Helmholtz equation. A key step in the development of NIM schemes is the Transverse Integration Process (TIP), which greatly simplifies the formulation. TIP transforms the partial differential equation (PDE) into a set of approximate ordinary differential equations (ODEs). The final scheme is constructed using the exact local solutions of these ODEs over each node. Employing exact ODE solutions enhances the accuracy of the method, which allows for the use of coarser computational grids in NIM. To obtain these exact solutions, we apply impedance boundary conditions instead of Dirichlet boundary conditions, which often lead to ill-posed problems. The use of impedance boundary conditions significantly improves the conditioning of the resulting matrix system and helps control its norm. The scheme is tested on the 2D Helmholtz equation for wave numbers $K \in [1, 500]$, demonstrating that the matrix norm remains bounded and consistently below 2.

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Shifted-penalty multigrid method for contact

Gabriele Marchi

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High-performance computing is essential for efficiently solving large-scale contact problems. Simulating such phenomena at engineering scale is often limited by computational resources, making it crucial to design algorithms that fully exploit modern hardware like multi-core CPUs and GPUs. Iterative solvers and preconditioners play a central role in this efficiency.

Monotone Multigrid (MMG) methods offer optimal complexity and robustness. In parallel, Penalty and Augmented Lagrangian methods handle over-constrained and fuzzy constraints effectively. Among these, the Shifted-Penalty method is notable for accurately enforcing constraints while remaining competitive with non-smooth techniques like the semi-smooth Newton method.

To combine the optimal complexity of MMG with the flexibility of shifted-penalty methods, we introduce the Shifted-Penalty Multigrid (SPMG) method. Designed from the ground up for GPU architectures, SPMG unifies nonlinear smoothing with constraint-aware multigrid strategies.

Our implementation uses matrix-free differential operators and memory-efficient semi-structured meshes to discretize elasticity equations. We present the SPMG algorithm with a focus on nonlinear smoothing and constraint coarsening.

We evaluate performance on the Grace-Hopper superchip of the CSCS Alps supercomputer. Emphasis is placed on single-node GPU performance, kernel design, and convergence behavior in simple contact scenarios. Finally, we demonstrate SPMG's scalability on large-scale problems with hundreds of millions of degrees of freedom.

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Machine learning-driven mesh adaptation and scientific computing at scale with MATLAB

Paolo Panarese

MathWorks

Machine learning (ML) and high-performance computing (HPC) are revolutionizing computational science, enabling efficient and scalable solutions for complex physical simulations. This poster presents a collection of workflows and applications in MATLAB that leverage ML-driven mesh refinement and agglomeration, as well as advanced scientific machine learning (SciML) and HPC techniques. Element-wise mesh refinement is guided by convolutional neural networks (CNNs), while graph neural networks (GNNs) and clustering algorithms enable high-quality agglomeration on general polygonal and polyhedral meshes. We further explore SciML approaches such as physics-informed neural networks (PINNs), which embed PDE constraints directly into custom loss functions, and architectures enforcing monotonicity, convexity, or Lipschitz continuity to ensure physically consistent solutions. MATLAB's extended Deep Learning and HPC toolboxes provide an integrated environment for rapid prototyping, visualization, and deployment of these workflows. Through practical case studies, the poster highlights the effectiveness and scalability of ML-enhanced mesh adaptation and SciML techniques in MATLAB, offering new capabilities for computational science and engineering applications.

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Increasing the efficiency of space-time multigrid for parabolic problems

Ausra Pogozelskyte

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Parallel-in-time methods allow us to parallelize partial differential equations not only in space but in time too. An example is the Space-Time Multigrid (STMG) algorithm, introduced by Gander and Neumüller, which has been shown to be highly efficient for parabolic problems.

In previous work, we applied Local Fourier Analysis (LFA) to optimize the smoother of STMG and increased the efficiency of the algorithm by at most a factor two. This analysis has been performed assuming that the problem is defined on a continuous grid.

In the multigrid framework, problems are solved using a hierarchy of grids that become progressively smaller, thus assuming that the coarsest grid is continuous is not realistic. In this poster, we show how a discrete analysis can further improve the efficiency of the algorithm, without any additional runtime costs.

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Simulating kidney hemodynamics with Navier-Stokes-Darcy equations

Fenfen Qi

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Understanding the hemodynamic alteration in renal diseases is important for treatment planning. This study employs numerical simulations to investigate the effects of pathological conditions on renal blood flows with patient-specific kidney geometry. We model the renal blood flow using coupled unsteady Navier-Stokes-Darcy equations discretized spatially with a stabilized mixed finite element method and temporally with a second-order backward differentiation formula, and the resulting nonlinear algebraic systems are solved efficiently using a Newton-Krylov-Schwarz algorithm. Based on the hemodynamic and structural characteristics, a two-level additive Schwarz preconditioner with a multiscale coarse preconditioner is constructed to accelerate the convergence. Through numerical experiments, we demonstrate the performance of the proposed algorithm for a realistic kidney and present results for different pathological features that show insight of the disease mechanism.

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An alternative to heterogeneous DD: the encounter between PDEs and imaging in cardiac hemodynamics

Rancesca Renzi

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Characterizing flow within the right heart (RH) is particularly challenging due to its complex geometries. A possible solution is to a consider an heterogeneous Domain Decomposition strategy for the Fluid-Structure Interaction arising between the muscular deformation and blood dynamics. This however requires to calibrate the structure problem and to deflate the geometry to receve a stress free configuration. For this reason, we consider here a pipeline for patient-specific simulations of RH hemodynamics which makes use of information about the muscular displacement coming from cineMRI medical images rather than a finite elasticity problem. We reconstruct the geometry and motion of the patient's right atrium, ventricle, and pulmonary and tricuspid valves. For this purpose, we develop a novel and flexible reconstruction procedure that integrates patient-specific tricuspid valve dynamics into a computational model, enhancing the accuracy of our RH blood flow simulations. We apply this approach to study the hemodynamics in a healthy RH and a repaired Tetralogy of Fallot RH with severe pulmonary regurgitation, as well as to assess the hemodynamic changes induced by the pulmonary valve replacement intervention.

Joint work with G.B. Luciani, G. Puppini, E. Xhelo, and C. Vergara.

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Do you precondition on the left or on the right?

Nicole Spillane

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The idea behind preconditioning is to accelerate a linear solver by providing it with an approximate inverse of the problem matrix. There are two main ways to precondition the problem $Ax = b$. Letting H be the preconditioner:

- either, $HAx = Hb$ is solved (left preconditioning),
- or, $AHu = b$ is solved and the solution is $x = Hu$ (right preconditioning).

Split preconditioning is also an option.

The goal of this poster is to present similarities and differences between left, right and split preconditioning. We also aim to start a conversation about whether there is a best choice and what your practices are when it comes to preconditioning.

Joint work with P. Matalon.

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Manifold parareal–a parareal algorithm for gradient flow

Yafei Sun

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We present a new parallel algorithm, *Manifold Parareal*, for accelerating the solution of the gradient flow model derived from solving a class of matrix inverse eigenvalue problems. We show that our proposed *Manifold Parareal* algorithm has a fast and robust convergence rate when using classical numerical schemes for the $\mathcal{F}$ and $\mathcal{G}$ propagators. Since the gradient flow model considered is characterized by a matrix-valued ODE defined on the Stiefel manifold, the solution inherently has an orthogonality property. Unfortunately, classical numerical schemes often fail to preserve this orthogonality in the final numerical solution. To address this issue, we also present several variants of *Manifold Parareal*: the key point lies in adopting some different retraction strategies during the solution process. This approach ensures that the orthogonality of the solution is not significantly compromised, thereby improving the accuracy of results and alleviating the restrictions on numerical schemes. Additionally, we are also exploring the application of *Manifold Parareal* to a special gradient flow–landing flow–which can be regarded as a modification of the gradient flow: by incorporating the direction that minimizes the orthogonal function $\mathcal{N}(Q) \triangleq \frac{1}{4}\left\|Q^\top Q - I\right\|^2$ into the original gradient flow model, the orthogonality of solution is naturally taken into account and retractions become unnecessary. Further detailed analyses of modified *Manifold Parareal* and landing flow are currently in progress.

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Additively preconditioned trust region strategies for machine learning

Ken Trotti

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We propose a new variant of the Additively Preconditioned Trust-Region Strategy (APTS) tailored for training neural networks. In this approach, APTS operates as a right-preconditioned trust-region method in which the preconditioner is built via an additive domain-decomposition scheme. Depending on the chosen decomposition, the domains can be either subsets of network parameters or partitions of the training dataset. By embedding APTS within the trust-region framework, we maintain its guarantee of global convergence toward a minimizer and eliminate the need for manual step-size selection, since each iteration's trust-region subproblem automatically determines an appropriate update magnitude. Through extensive numerical experiments, we benchmark our method against popular optimizers such as SGD, Adam and L-BFGS, as well as against a standard trust-region solver, highlighting where our APTS variant excels and where it faces limitations.

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Numerical studies of the Brugada syndrome with a parallel bidomain model

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Cardiac arrhythmia occurs when the electrical signals that tell the heart to beat don't work properly. In this study, we focus on a particular type of arrhythmia, Brugada Syndrome, which is a genetic disease that predisposes individuals to life-threatening arrhythmias with elevated risk of ventricular fibrillation. A three-dimensional computational model of the ventricles is developed to investigate how the location of an abnormal substrate influences the initiation and maintenance of reentry, a mechanism where abnormal electrical circuits in the heart can initiate arrhythmias. Using a patient-specific heart geometry and a framework that combines finite element methods, implicit-explicit time stepping, and a preconditioned Krylov subspace solver, action potential propagation is simulated in the ventricles with a single substrate placed in various regions of the right ventricle. The parallel performance of the proposed algorithm is studied, including variations in solver parameters and strong scaling on computers with up to 2048 cores.

Joint work with Y. Jiang, T. Ma, X. Wang, S. Scacchi, L.F. Pavarino, R. Chen, and X.-C. Cai.

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One-level integrated additive domain decomposition solver for LOD coarse problem for Helmholtz problem with high wave number

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The Helmholtz equation is of great interest in many important applications, and to solve it efficiently is notoriously difficult. In the previous work we designed a coarse space based on the LOD method for a hybrid two-level DDM, and proved its optimality. When the wave number grows, to get the coarse solution will become the main computational cost. We constructed a novel one-level additive DDM for solving the coarse problem in the two-level framework, and proposed some propositions for its well-posedness, efficiency and convergence.

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